LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2013**

MT 1502 - ALGEBRA AND CALCULUS - I

Date: 09/05/2013 Dept. No. Max. : 100 Marks

Time: 1:00 - 4:00

PART A **(10 × 2 = 20)**

ANSWER **ALL** QUESTIONS

1. Write the nth derivative of $\log(ax + b)$.
2. Show that, in the parabola $y^2 = 4ax$, the subtangent at any point is double the abscissa.
3. Write the condition for maxima and minima of a function of two variables.
4. Find the radius of curvature at $(1, \frac{1}{2})$ on the curve $2y = x(1 - x + x^2)$.
5. Find the co-ordinates of the centre of curvature of the curve $xy = 2$ at the point (2, 1).
6. Form the quadratic equation having $\sqrt{5} - 1$ as a root.
7. Solve $2x^3 - 15x^2 + 46x - 42 = 0$, given that $3 - i\sqrt{5}$ is a root.
8. State Newton's theorem on the sum of the powers of the roots.
9. State Descarte's rule of signs for negative roots.
10. Show that the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.

PART B **(5 × 8 = 40)**

ANSWER ANY **FIVE** QUESTIONS

11. If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)y_2 - xy_1 + m^2 y = 0$ and
$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

12 . Find the angle at which the radius vector cuts the curve $\frac{l}{r} = 1 + e\,cos\theta$.

13. Find the maximum and minimum values of the function $(x, y) = xy + \frac{1}{x} + \frac{1}{y}$.

14. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta)$, $y = a(1 - cos\theta)$ is $4a \cos\frac{\theta}{2}$.

15. Find the asymptotes of $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$.

16. Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in Arithmetic Progression if $2p^3 - 9pq + 27r = 0$.

17. Calculate the sum of the cubes of the roots of the equation $x^4 + 2x + 3 = 0$.

18. Solve $x^3 - 6x - 9 = 0$ by Cardon's method.

PART C **(2 × 20 = 40)**

ANSWER ANY **TWO** QUESTIONS

19. (a). Solve $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$. (10)

(b). Find the angle of intersection of the cardioids

$r = a(1 + cos\theta)$ and $r = b(1 - cos\theta)$. (10)

20. If $u = a^3 x^2 + b^3 y^2 + c^3 z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of u.

21. Find the evolute of the cycloid $x = a(\theta - sin\theta)$; $y = a(1 - cos\theta)$.

22. Show that the equation $x^3 - 3x + 1 = 0$ has a root between 1 and 2 and calculate it to two places of decimals. (10)

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