LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2013**

MT 1503 - ANALYTICAL GEOMETRY OF 2D,TRIG. & MATRICES

Date: 11/05/2013 Dept. No. Max. : 100 Marks

Time: 1:00 - 4:00

PART-A

Answer ALL the questions: **10 x 2=20**

1. Write the coefficient of $\cos^n \theta$ in the expansion of $\cos n\theta$.
2. Expand $\cos^5 \theta$ in terms of cosines of multiples of θ.
3. Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x.$
4. Find the value of $Log(1-i).$
5. Find the eigen values of the matrix $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$.
6. State Cayley-Hamilton theorem.
7. Find the condition for the lines $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$ to be conjugate.
8. Define conjugate diameter of the ellipse.
9. If e_1 and e_2 are the eccentricities of a hyperbola and its conjugate then prove that $e_1^{-2} + e_2^{-2} = 1.$
10. Define Polar Co-ordinates.

PART –B

Answer any FIVE questions: **5 x 8=40**

11. Prove that the equation $\frac{ah}{\cos\theta} - \frac{bk}{\sin\theta} = a^2 - b^2$ has four roots.
12. Evaluate $\lim_{\theta \to 0} \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$.
13. If $\cosh u = \sec\theta$ then prove that $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$.
14. Separate $\tan^{-1}(x + iy)$ into real and imaginary parts.
15. Calculate A^4 when $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$.

16. Find the locus of the midpoints of chords of the parabola which subtend a right angle at the vertex of the parabola.
17. Prove that the conjugate lines through a focus of an ellipse are at right angles.
18. The asymptotes of a hyperbola are parallel to $2x+3y=0$ and $3x-2y=0$. Its centre is at $(1,2)$ and it passes through the point $(5,3)$. Find its equation and its conjugate.

PART-C

Answer any TWO questions: **2 x 20=40**

19. a) Prove that $\sin^3\theta\cos^5\theta=\frac{-1}{2^7}[\sin 8\theta+2\sin 6\theta-2\sin 4\theta-6\sin 2\theta]$.

    b) Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$.

20. a) If $\cos(x+iy)=\cos\theta+i\sin\theta$ then prove that $\cos 2x+\cosh 2y=2$.

    b) Reduce $(\alpha+i\beta)^{x+iy}$ to the form $A+iB$.

21. Diagonalise the matrix $\begin{bmatrix}2 & 2 & 0\\ 2 & 1 & 1\\ -7 & 2 & -3\end{bmatrix}$.

22. a) Trace the curve $\frac{10}{r}=3\cos\theta+4\sin\theta+5$.

    b) A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose centre C meets the circle $x^2+y^2=a^2+b^2$ at Q and Q'.Prove that CQ and CQ' are conjugate diameters of the ellipse.

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