# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **CHEMISTRY**

**FIRST** SEMESTER – **APRIL 2016**

**MT 1102 - MATHEMATICS FOR CHEMISTRY**

Date: 05-05-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

## SECTION-A

**Answer ALL questions.** **(10 x 2 = 20)**

1. For what value of $x$ is the curve $y = 3x^2 - 2x^3$ convex upwards.
2. Find $\frac{dy}{dx}$ if $y = (2x^3 + 4)^2$.
3. Evaluate $\int x^2 cosx^3 dx$.
4. Integrate $\frac{x^2}{x+2} dx$.
5. Write the series expansion of log 3.
6. Show that $\log\left(\frac{a+x}{a-x}\right) = \frac{2ax}{a^2+x^2} + \frac{1}{3}\left(\frac{2ax}{a^2+x^2}\right)^3 + \frac{1}{5}\left(\frac{2ax}{a^2+x^2}\right)^5 + \ldots$
7. Prove that $\cosh^2 x - \sinh^2 x = 1$
8. Define Fourier series.
9. State any one property of Arithmetic Mean.
10. Write the mean of the Binomial Distribution.

## SECTION B

**Answer any FIVE questions:** **(5 x 8 = 40)**

11. Evaluate $\int x^2 sin3x\, dx$ using Bernoulli's formula.
12. If $x(1+y)^{\frac{1}{2}} + y(1+x)^{\frac{1}{2}} = 0$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.
13. Find the maximum value of $\frac{logx}{x}$ for positive values of $x$.
14. Solve $\left(D^3 + 2D^2 + D\right)y = e^{2x}$.
15. Sum the series $\frac{1.4}{5.10} - \frac{1.4.7}{5.10.15} + \frac{1.4.7.10}{5.10.15.20} \ldots$.
16. Solve $p + q = \sin x + \sin y$.
17. Expand $\cos 6\theta$ in terms of $\sin\theta$.
18. Calculate the mean and standard deviation for the following table giving the age distribution of 542 members.

| Age in years | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 |
|---|---|---|---|---|---|---|---|
| Num. of members | 3 | 61 | 132 | 153 | 140 | 51 | 2 |

## SECTION C

**Answer any TWO questions:** **(2 x 20 = 40)**

19. (a) From a solid sphere, matter is scooped out so as to form a conical cup, with the vertex of the cup on the surface of the sphere. Find when the volume of the cup is a maximum. **(10)**

(b) Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$. **(10)**

20. (a) Evaluate $I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx$. **(10)**

(b) Prove that $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}} + (\cos x)^{\frac{3}{2}}} dx = \frac{\pi}{4}$. **(10)**

21. (a) If a, b, c are three consecutive integers, show that

$$\log_e b = \frac{1}{2}\log_e a + \frac{1}{2}\log_e c + \left(\frac{1}{2ac+1}\right) + \frac{1}{3}\left(\frac{1}{2ac+1}\right)^3 + \ldots$$ **(10)**

(b) Find the real and imaginary parts of $\tan^{-1}(x+iy)$. **(10)**

22. (a) Determine the Fourier series expansion of $x + x^2$ in the interval $(-\pi, \pi)$ and hence determine the sum of series $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots$ **(10)**

(b) Two unbiased dice are thrown. Find the probability that:

(i) Both the dice show the same number,

(ii) The first die shows 6,

(iii) The total of the numbers on the dice is 8. **(10)**

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