LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2016**

MT 1500 – ALGEBRA, ANALY. GEO., CALCULUS & TRIGONOMETRY

Date: 02-05-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A **(10 x 2 = 20)**

Answer ALL questions

1. Write down the n^{th} derivative of $y = \log(ax+b)$.
2. Find the polar subtangent and polar subnormal of the curve $r = a\theta$.
3. Write the formula for the radius of curvature when the pedal equation of the curve is given.
4. Define evolute of a curve.
5. If α, β are the roots of the equation $ax^2 + bx + c = 0$, form the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$.
6. Form a rational cubic equation, given that $1, 3-\sqrt{-2}$ are the two given roots.
7. Prove that $\cosh 2x = \frac{1+\tanh^2 x}{1-\tanh^2 x}$.
8. Evaluate $\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta}$.
9. Find the pole of $3x + 8y - 24 = 0$ with respect to $9x^2 + 16y^2 = 144$.
10. Find the equation of the chord of the parabola $y^2 = 4ax$ having $(1,1)$ as its midpoint.

PART – B **(5 x 8 = 40)**

Answer any FIVE questions

11. If $y = x^2 e^x$ prove that $y_n = \frac{1}{2}n(n-1)y_2 - n(n-2)y_1 + \frac{1}{2}(n-1)(n-2)y$.
12. Using Lagrange's multipliers method find the maximum and minimum values of $f(x,y) = x^2 - y^2$ subject to $x^2 + y^2 = 1$.
13. Find the radius of curvature at the point θ of the curve $x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta)$.
14. Solve the equation $x^3 - 19x^2 + 114x - 216 = 0$, given that the roots are in GP.
15. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, prove that $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha) = r - pq$.
16. Expand $\sin^3\theta\cos^4\theta$ in terms of multiples of θ.
17. Show that the eccentric angles at the extremities of a pair of semi conjugate diameters of an ellipse differ by a right angle.
18. Find the angle between the asymptotes of the hyperbola. Show that the product of the perpendiculars drawn from any point on a hyperbola to its asymptotes is constant.

PART– C **(2 x 20 = 40)**

Answer Any TWO Questions

19. a) If $y=\left(x+\sqrt{1+x^2}\right)^m$ then prove that $(1+x^2)\,y_{n+2}+(2n+1)x\,y_{n+1}+(n^2-m^2)y_n=0.$

b) Find the angle of intersection of the curves

$$r=\frac{a}{1+\cos\theta} \text{ and } r=\frac{b}{1-\cos\theta}.$$ (10 +10)

20. a) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$

b) Solve the reciprocal equation $6x^5+11x^4-33x^3-33x^2+11x+6=0$. (10 +10)

21. a) Calculate to two places of decimals, the positive root of the equation $x^3+24x-50=0$ by Horner's method.

b) Separate into real and imaginary parts of $\tan^{-1}(x+iy)$. (12 + 8)

22. a) Find the sum of the series $\sum_{n=1}^{\infty}\frac{\sin n\theta}{2^n}$.

b) A Rectangular hyperbola whose centre is C is cut by any circle of radius r in the four points P,Q,R,S. Prove that $CP^2+CQ^2+CR^2+CS^2=4r^2$. (10 +10)

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