LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2016**

MT 1819 - PROBABILITY THEORY & STOCHASTIC PROCESSES

Date: 30-04-2016 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Section – A

Answer all the questions **10 x 2 = 20 marks**

1. Write the sample space for casting two fair dice simultaneously.
2. Write two properties of distribution function..
3. If $f(x) = 2e^{-2x}$, $0 < x < \infty$, zero elsewhere , find E(X).
4. If P(A) = 1/3 , P(B) = 1/5 and $P(A \cap B) = 1/9$ find (i) $P(B \mid A)$ and (ii) $P(B \mid A^C)$.
5. Define a standard normal distribution
6. Write the p.d.f. and M.G.F. of gamma distribution with two parameters.
7. Define convergence in probability.
8. Write the sufficient conditions for an estimator to be consistent.
9. Write about Type I and Type II errors in testing of hypothesis.
10. Define communication of states of a Markov chain.

Section – B

Answer any Five questions **5x8 = 40 marks**

11. State and prove addition theorem on probability for n events.
12. If X has the p.d.f. $f(x) = (4-x) / 16$, $-2 < x < 2$, zero elsewhere , find the first four central moments.
13. Derive the moment generating function of normal distribution .
14. (a) Define binomial distribution.
    (b) Show that under certain conditions binomial tends to Poisson. (2+6)
15. State and prove Cramer-Rao inequality .
16. If $X_1, X_2, \ldots X_n$ is a random sample from $N(\mu, \sigma^2)$ find the maximum likelihood estimators for (a) μ when σ^2 is known (b) σ^2 when μ is known.
17. The heights (in inches) of 12 persons of a given locality are known to be 72 , 69, 64, 70 , 63 , 70 , 72 , 66 ,66 , 68. Test whether the average height is greater than 68 inches at 1% level of significance.
18. (a) Write about periodicity and recurrence of Markov chain.
    (b) Show that communication is an equivalence relation.

(4+4)

Section – C

Answer any two questions **2 x 20 = 40 marks**

19. (a) State and prove Chebyshev's inequality.
    (b) If X has the p.d.f. $f(x) = 6x(1-x)$, $0 < x < 1$, zero elsewhere , find mean and variance.
    (c) If X has the p.d.f. $f(x) = 1/\pi(1+x^2)$, $-\infty < x < \infty$, find the median of the distribution. **(8+8+4)**
20. (a) Find the first and second central moments of Beta distribution of first kind.
    (b) Let X be $N(\mu , \sigma^2)$ so that P (X < 89) = 0.90 and P(X < 94) = 0.95. Find μ and σ^2 . **(10+10)**
21. (a) Let X_1 and X_2 have the joint pdf $f(x_1, x_2) = x_1 + x_2$, $0< x_1< 1, 0< x_2< 1$, zero elsewhere .Find the conditional mean and variance of X_2 given $X_1 = x_1$, $0 < x_1 < 1$.
    (b) State and prove Boole's inequality. (10+10)
22. (a) Fit a Poisson distribution to the following data and test the goodness of fit at $\alpha = 0.05$:

No. of errors/ page (x) :	0	1	2	3	4	5
No. of pages (f) :	125	78	30	10	5	2

    (b) Derive $P_n(t)$ for Poisson process clearly stating the postulates. **(10 + 10)**

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