

**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034****B.Sc. DEGREE EXAMINATION – MATHEMATICS****SECOND SEMESTER – APRIL 2016****MT 2501/MT 2500 – ALGEBRA, ANAL.GEO & CALCULUS - II**

Date: 21-04-2016

Dept. No.

Max. : 100 Marks

Time: 01:00-04:00

PART-A**Answer ALL the questions****(10 x 2=20 marks)**

1. Evaluate $\int_0^1 \left(x^2 - 3x^{1/2} + \frac{1}{x^2} \right) dx$.
2. Evaluate $\int_0^{\pi/2} \sin^7 x dx$.
3. Solve $\sqrt{1+y^2} + \sqrt{1+x^2} \frac{dy}{dx} = 0$.
4. Solve $(D^2 + 4D + 4)y = 0$.
5. Show that $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$ is convergent.
6. Test for convergency of the series $\sum_{n=0}^{\infty} \frac{n^3 + 1}{2^n + 1}$.
7. Expand $(1-x)^{-2}$ and $(1-x)^{-1/2}$.
8. Expand $\frac{e^x + e^{-x}}{2}$ and $\log(1+x)$.
9. Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$.
10. Find the radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z - 15 = 0$.

PART-B**Answer any FIVE questions****(5 x 8=40 marks)**

11. Evaluate $\int_0^{\pi/2} \frac{(\sin x)^3}{(\sin x)^3 + (\cos x)^3} dx$.
12. Evaluate $\int (\log x)^3 x^4 dx$.
13. Solve $(D^2 + 2D + 5)y = xe^x$.
14. Solve $\frac{d^2y}{dx^2} + y = \sec x$.
15. Test for convergency and divergency the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \dots$
16. Sum the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots$
17. Show that $\frac{5}{1.2.3} + \frac{7}{3.4.5} + \frac{9}{5.6.7} + \dots = 3 \log 2 - 1$.
18. Find the equation of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$ and $(5, 3, -3)$.

PART-C

Answer any TWO questions

(2 x 20=40 marks)

19. (a) Evaluate $\int \frac{x^3 + 1}{\sqrt{1-x^2}} dx$. (10)

(b) Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x dx$. (10)

20. (a) Solve $xyp^2 + (3x^2 - 2y^2)p - 6xy = 0$. (10)

(b) Solve $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$. (10)

21. (a) Show that $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \dots$ (10)

(b) Sum the series $1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \dots$. (10)

22. (a) Examine the convergence of the series $\sum \frac{(n+1)(n+2)\dots(n+n)}{n^n}$. (10)

(b) Prove that the lines $\frac{x+1}{-3} = \frac{y+10}{8} = \frac{z-1}{2}$; $\frac{x+3}{-4} = \frac{y+1}{7} = \frac{z-4}{1}$ are coplanar find also their point of intersection and plane through them. (10)

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