LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2016**

MT 2806 - PARTIAL DIFFERENTIAL EQUATIONS

(6th batch)

Date: 16-04-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

ANSWER ALL QUESTIONS

1. a) (i) Find the general integral of $z = xy + f(x^2 + y^2)$.

OR

(ii) Discuss the partial differential equation $f(z,p,q) = 0$ and find the Complete integral of $p(1+q^2) = q(z-1)$. **(5 Marks)**

b) (i) Obtain the condition of compatibility of $f(x,y,z,p,q) = 0$ and $g(x,y,z,p,q) = 0$

(ii) Use Charpit's method to solve $p = (z + qy)^2$ **(7+8 Marks)**

OR

(iii) Show that $xp - yq = x$ and $x^2p + q = xz$ are compatible and find the solution.

(iv) Use Jacobi's method to solve $p^2x + q^2y = z$. **(7 +8 Marks)**

2. a) (i) Reduce $U_{xx} + 4U_{xy} + 4U_{yy} + U_x = 0$ to a canonical form.

OR

(ii) Solve $(D^2 - 2DD' + D'^2)z = x^3$ **(5Marks)**

b) (i) Find the solution of the equation $\nabla_1^{\,2} z = e^{-x}\cos y$, which tends to zero as $x \to \infty$ and has the value $\cos y$ when $x = 0$. **(15 Marks)**

OR

(ii) Obtain the canonical form of the elliptic partial differential equation.

(iii) Solve $(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = e^{x+y}$ **(8+7 Marks)**

3. a) (i) Obtain the Laplace equation.

OR

(ii) Obtain the Poisson's equation. **(5 Marks)**

b) (i) Solve a two dimensional Laplace equation subject to the boundary conditions: $u(x, 0) = 0$, $u(x, a) = 0$, $u(x, y) \to 0$ as $x \to \infty$ when $x \geq 0$ and $0 \leq y \leq a$.

OR

(ii) State and prove Interior Dirichlet Problem for a Circle. **(15 Marks)**

4. a) (i) Derive reduction of one dimensional wave equation to canonical form and its equation.

OR

(ii) Discuss the Transmission Line problem. **(5 Marks)**

b) (i) Obtain the solution of Diffusion equation in spherical coordinates.

OR

(ii) A transmission line 1000 km long is initially under steady state conditions with potential 1300 volts at the sending end $x = 0$ and 1200 volts at the receiving end $x = 1000$. The terminal end of the line is suddenly grounded but the potential at source is kept at 1300 volts. Assuming the inductance and leakance to be negligible, find the potential $v(x,t)$. **(15 Marks)**

5. a) (i) If $f(z)$ be analytic for $Re(z) \geq \gamma$ where γ is a real constant greater than zero, then for $Re(z_0) > \gamma$, then $f(z_0) = \frac{1}{2\pi i}\int_{\gamma-i\beta}^{\gamma+i\beta} \frac{f(z)dz}{z-z_0}$.

OR

(ii) Show that the Green's function $G(\vec{r},\vec{r}')$ has the symmetry property. **(5 Marks)**

b.(i) Solve the initial value problem by using Laplace transform method $k\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < l,\ 0 < t < \infty$, subject to the condition $u(0,t) = 0, u(l,t) = g(t)$, $0 < t < \infty$ and $u(x,0) = 0, 0 < x < l$.

OR

(ii) Use Green's function technique to solve Dirichlet's problem for an infinity space.

(15 Marks)

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