LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – **APRIL 2016**

MT 2812 - PARTIAL DIFFERENTIAL EQUATIONS

(14th & 15th BATCHES)

Date: 16-04-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) Show that the equations $p^2 + q^2 = 1$, $(p^2 + q^2)x = pz$ are compatible and solve them. **(5)**

OR

(b) Using Charpit's method, solve $(p^2 + q^2)^n(qx - py) = 1$. **(5)**

(c) Find the complete integral for the following equations using Jacobi's method:
(i) $p^2x + q^2y = z$ (ii) $xpq + yq^2 = 1$ (iii) $p = (z + qy)^2$. **(4 + 5 + 6)**

OR

(d) Find the characteristic of the partial differential equation $z = p^2 - q^2$ and find the integral surface which passes through the parabola $4z + x^2 = 0, y = 0$. **(15)**

2. (a) Define self adjoint operator. Prove that (i) $L(u) = u_{xx} + u_{yy}$ (ii) $L(u) = c^2u_{xx} - u_{tt}$ are self adjoint. **(5)**

OR

(b) Solve $(2D^2 - 5DD' + 2D')z = 24(y - x)$. **(5)**

(c) Explain Riemann's method to solve linear hyperbolic partial differential equations. **(15)**

OR

(d) Reduce the equations $u_{xx} + y^2u_{yy} = y$ to canonical form and solve. **(15)**

3. (a) Derive Laplace equation. **(5)**

OR

(b) Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r, \theta)$. **(5)**

(c) Obtain the solution of heat conduction equation in spherical polar coordinates. **(15)**

OR

(d) Solve the Cauchy problems, described by the inhomogenous wave equation
$\frac{\partial^2 u}{\partial x^2} - c^2\frac{\partial^2 u}{\partial y^2} = f(x, t)$ subject to initial conditions $u(x, 0) = \eta(x)$, $\frac{\partial u}{\partial t}(x, 0) = v(x)$. **(15)**

4. (a) Solve the heat conduction equation $k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ $-\infty < x < \infty$, $t > 0$ subject to the initial and boundary conditions $u(x, t)$ and $\frac{\partial u}{\partial x}(x,t) \to 0$ as $|x| \to \infty$, $u(x, 0) = f(x)$, $-\infty < x < \infty$. **(5)**

OR

(b) Determine the Green's function for the Dirichlet problem for a circle. **(5)**

(c) Use Laplace transform method, to solve the initial value problem $\frac{\partial^2 u}{\partial x^2} = k\frac{\partial u}{\partial t}$, $0 < x < l$, $0 < t < \infty$ subject to the conditions $u(0, t) = 0$, $u(l, t) = g(t)$, $0 < t < \infty$ and $u(x, 0) = 0$, $0 < x < l$. **(15)**

OR

(d) State and prove Helmholtz Theorem. **(15)**

5. (a) Find the solution of the Volterra integral equation
$y(x) = sinx + 2\int_0^x \cos(x-t)y(t)dt$. **(5)**

OR

(b) Determine the resolvent kernels for the Kernel $K(x, t) = e^{x+1}$; $a = 0$, $b = 1$. **(5)**

(c) Find the solution of Fredholm integral equation of second kind by the method of successive substitutions. **(15)**

OR

(d) State and prove Hilbert- Schmidt theorem. **(15)**

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