Date: 06-05-2016   Dept. No.   Max. : 100 Marks
Time: 09:00-12:00

PART A

Answer ALL the questions   (10 x 2 =20)

1. If $y = e^{ax}$, find y_n.

2. Find the polar subtangent and subnormal of the curve $r = e^{\theta \cot\alpha}$.

3. Prove that $\dfrac{e-1}{e+1} = \dfrac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\cdots\infty}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\cdots\infty}$.

4. Find the rank of the matrix $A = \begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & -4 \\ -3 & 1 & -2 \end{pmatrix}$.

5. Find the Laplace transform of $3t - 2e^{-t}$.

6. Find $L^{-1}\left(\dfrac{1}{(s+1)^2+1}\right)$.

7. Write down the expansion of $\sin\theta$ and $\cos\theta$ in a series of ascending powers of θ.

8. Prove that $\cosh^2 x - \sinh^2 x = 1$.

9. What is the probability of getting a head and a tail when two coins are tossed simultaneously.

10. Write down the probability mass function for the Binomial distribution.

PART B

Answer any FIVE questions   (5 x 8 =40)

11. Find the nth differential coefficient of $x^2 \log x$.

12. Find the angle of intersection of the cardioids $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.

13. Show that $\log(x + 2h) = 2\log(x + h) - \log x - \left[\dfrac{h^2}{(x+h)^2} + \dfrac{h^4}{2(x+h)^4} + \dfrac{h^6}{3(x+h)^6} + \cdots\right]$.

14. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

15. Find (i) $L\left(\dfrac{e^{-3t}-e^{-4t}}{t}\right)$ (ii) $L(t \sin 2t)$.

16. Write down the expansion of $\cos 6\theta$ in terms of $\cos\theta$.

17. Two unbiased dice are thrown. Find the probability that (i) both the dice show the same number (ii) the first die shows 6 (iii) the total of the numbers on the dice is 8 (iv) the total of the numbers on the dice is 13.

18. An insurance company insures 4,000 people against loss of both eyes in a car accident. Based on previous data, the rates were computed on the assumption that on the average 10 persons in 1,00,000 will have car accident each tear that result in this type of injury. What is the probability that more than 3 of the insured will collect on their policy in a given year?

1

PART C

Answer any TWO questions (2 x 20 =40)

19. (a) Find the lengths of the subtangent and subnormal at (a, a) on the cissoid $y^2 = \frac{x^3}{2a-x}$.

(b) Find the maximum and minimum values of the function $f(x) = 2x^3 - 3x^2 - 36x + 10$.

(10+10)

20. (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.

(b) Find the sum to infinity of the series $1 + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + \cdots \infty$.

(12+8)

21. (a) Solve the equation $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$, given that $y = 0, \frac{dy}{dt} = 2$ when $t = 0$.

(b) Expand $sin^3\theta cos^5\theta$ in a series of sines of multiples of θ.

(12+8)

22. (a) If $sin(A + iB) = x + iy$, prove that (i) $\frac{x^2}{cosh^2 B} + \frac{y^2}{sinh^2 B} = 1$ (ii) $\frac{x^2}{sin^2 A} - \frac{y^2}{cos^2 A} = 1$.

(b) Calculate the mean and standard deviation for the following table giving the age distribution of 542 members:

Age(in years)	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of members	3	61	132	153	140	51	2

(8+12)

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