



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – APRIL 2016

MT 3501/MT 3500 – ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date: 28-04-2016

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

PART-A

Answer ALL the Questions:

[10x2=20marks]

1. Evaluate $\int_0^a \int_0^b \int_0^c dx dy dz$.
2. Evaluate $\int_0^{-\pi/2} \sin^5 \theta \cos^3 \theta d\theta$.
3. Solve $pq = 1$.
4. Form PDE by eliminating the arbitrary function $z = f(x^2 + y^2)$.
5. Prove that $\text{div } \vec{r} = 3$ where \vec{r} is the position vector of the point (x, y, z) .
6. State Green's theorem.
7. Evaluate $L [te^{-5t}]$.
8. Evaluate $L^{-1} \left(\frac{1}{(s+2)^2 + 16} \right)$.
9. State Fermat's theorem.
10. Find the number and sum of all divisors of 360.

PART - B

Answer any FIVE questions:

[5x8=40 Marks]

11. Evaluate $\iint xy dx dy$ over the region in the positive quadrant for which $x+y=1$.
12. Prove that $\sqrt{n+1} = n\sqrt{n}$.
13. Solve $x^2 p + y^2 q = z^2$.
14. a) Solve $\frac{\partial^2 z}{\partial x \partial y} = x + y$.
 b) Solve $p^2 + q^2 = x^2 + y^2$. (4 + 4)
15. Evaluate $L \left(\frac{1-\cos t}{t} \right)$.
16. Evaluate $L^{-1} \left[\frac{1}{s(s+1)(s+2)} \right]$.
17. If $\vec{F} = 2xyz\vec{i} + x^2z\vec{j} + x^2y\vec{k}$. Find the scalar potential .
18. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

PART – C

Answer any TWO questions:

[2x20=40 Marks]

19. a) Prove that $\beta(m, n) = \beta(m, n)$

b) Evaluate $\int_0^1 x^2 e^{-x} dx$ (15+5)

20. a) Solve $(z^2 - 4y) dx - 2z dy = 0$

b) Solve $(y^2 - x^2) dx + 2xy dy = 0$ (10+10)

21. a) Verify the Divergence theorem for $\vec{f} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over cube $x=0, x=1, y=0, y=1, z=0$ and $z=1$.

b) State and Prove Wilson's theorem. (10+10)

22. Use Laplace transform to Solve the equation $\frac{dy}{dx} + y = 2e^{2x}$ given that $y(0) = -3, y(1) = 5$.

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