LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2016**

MT 3810 - TOPOLOGY

Date: 25-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all questions. Each question carries 20 marks.

I a)1) Let X be a complete metric space and let Y be a subspace of X. Prove that Y is closed when Y is complete.

OR

a)2) Let X and Y be metric spaces and f a mapping of X into Y. Prove that f is continuous at x_o if $x_n \to x_o \Rightarrow f(x_n) \to f(x_o)$ **(5)**

b)1) If a convergent sequence in a metric space has infinitely many distinct points then prove that its limit point is a limit point of the set of points of the sequence.

b)2) State and prove Cantor's intersection theorem. **(9+6)**

OR

c)1) If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X, then prove that there exists a point in X which is not in any of the A_n's.

c)2) Let X be a metric space, let Y be a complete metric space and let A be a dense subspace of X. If f is uniformly continuous mapping of A into Y then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y. **(6+9)**

II a)1) Prove that every separable metric space is second countable.

OR

a)2) State and prove Lindelof's theorem. **(5)**

b)1) Prove that any closed subspace of a compact space is compact.

b)2) State and prove Heine-Borel theorem. **(6+9)**

OR

c)1) Prove that any continuous image of a compact space is compact.

c)2) Prove that a topological space is compact if every class of subbasic closed sets with the finite intersection property has non-empty intersection. **(5+10)**

III a)1) Prove that the product of any non-empty class of Hausdorff spaces is a Hausdorff space.

OR

a)2) Prove that every compact Hausdorff space is normal. **(5)**

b) Let X be a normal space and let A and B be disjoint closed subspaces of X. If [a,b] is any closed interval on the real line, then prove that there exists a continuous real function f defined on X, all of whose values lie in [a,b] such that f(A) =a and f(B) = b. **(15)**

OR

c)1) Prove that a one-to-one continuous mapping of a compact space into a Haudorff space is a homeomorphism.

c)2) State and prove Tietze Extension theorem. **(5+10)**

IV) a)1) Prove: A topological space X is disconnected $\Leftrightarrow$ there exists a continuous mapping of X onto the discrete two points space $\{0,1\}$.

OR

a)2) Prove that a subspace of the real line R is connected if it is an interval. **(5)**

b) Prove that the product of any non-empty class of connected spaces is connected and as its application, show that all finite-dimensional Euclidean and unitary spaces are connected. **(15)**

OR

c) If X is an arbitrary topological space, then prove the following:

(i) Each point in X is contained in exactly one component of X

(ii) Each connected subspace of X is contained in a component of X

(iii) A connected subspace of X which is both open and closed is a component of X and

(iv) Each component of X is closed. **(15)**

V)a)1) Quoting the lemmas required, state Stone-Weierstrass theorem.

OR

a)2) Prove that X_∞ is Hausdorff. **(5)**

b) State and prove Weierstrass approximation theorem.

OR

c) Proving the two required lemmas, state and prove the Extended Stone-Weierstrass theorem for (X,R).

(15)

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