LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2016**

MT 5406 - COMBINATORICS

Date: 25-04-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

SECTION-A

Answer all the questions. **(10×2=20)**

1. Define the falling factorial polynomial
2. Find the number of ways of assigning 15 identical posters to 18 dormitories so that no dormitory receives more than 1 poster.
3. State the multinomial theorem.
4. Define Derangement and write the formula to find the number of derangements of m distinct objects D_m.
5. Find the sequences of the ordinary generating functions $2x^2(1-x)^{-1}$ and $(3+x)^3$.
6. Find the coefficient of x^{27} in $(x^4+x^5+x^6+\cdots)^5$ and in $(x^4+2x^5+3x^6+\cdots)^5$.
7. Write the exponential generating functions of the following functions:
   a) $\{1,2,3,0,0,0\ldots\}$
   b) $\{0,0,1,1,1,1,\ldots\}$
8. Define the Stirling number of the second kind.
9. Calculate $per\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$.
10. Evaluate $\phi(3528)$.

SECTION-B

Answer any five questions. **(5×8=40)**

11. Derive recurrence formula for Stirling number of the second kind. Also construct the table of stirling numbers of the second kind when n=7 and m=7.
12. Define the rising factorial $[m]^n$ and prove that the number of distribution of n distinct objects into m distinct boxes with the order of objects in each box is significant and empty boxes are permitted is the rising factorial.
13. a) Use the concept of generating function to prove the Pascal's Identity.
    b) Find the ordinary generating function of the sequence $\{(n+r-1)C_{n-1}\}$ by differentiating the geometric series. **(3+5)**
14. Derive the formula to find the sum of first n natural numbers using its recurrence formula given by $a_n - a_{n-1} = n, n \geq 1$.
15. a) A box contains many identical red, blue, white and green marbles. Find the ordinary generating function corresponding to the problem of finding the number of ways of choosing r marbles from the box such that the sample does not have more than 2 red, more than 3 blue, more than 4 white and more than 5 green and hence find the number of ways of choosing 10 marbles from the box containing the mentioned sample.
    b) Find the number of ways of forming a committee of 9 people drawn from 3 different parties so that no party has an absolute majority in the party. **(5+3)**

16. Find the rook polynomial for the given chess board *C*:

17. State and prove Sieve's formula and hence find the number of positive integers less than 601 that are not divisible by 3 or 5 or 7.

18. State and prove Burnside Frobenius theorem.

SECTION-C

Answer any two questions **(2×20=40)**

19. a) If m and n are positive integers, prove that the equation $x_1 + x_2 + \cdots + x_m = n$ has exactly $\frac{[m]^n}{n!}$ solutions in nonnegative integers x_k.

b) In a town council there are 10 democrates and 11 republicans. There are 4 women among the democrats and 3 women among the republicans. Find the number of planning committee of eight councellors such that of equal number of men and women and equal number of member of both committee. **(14+6)**

20. State and prove the Ménage problem.

21. a) Find the rook polynomial for a 2×2 Chess board by the use of expansion formula.

b) An executive attending a week long seminar has 5 suits of different colors. On Mondays she does not wear blue or green, on Tuesdays she does not wear red or green, on Wednesdays she does not wear blue or white or yellow, on Fridays, she does not wear white. How many ways can she does without repeating a color for the seminar?

(6+14)

22. a) Prove the multinomial theorem and hence find the coefficient of $x_1^2 x_3 x_4^3 x_5^4$ in the expression $(x_1 + x_2 + x_3 + x_4 + x_5)^{10}$.

b) Show that 97 is the twenty-fifth prime. **(12+8)**

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