LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2016**

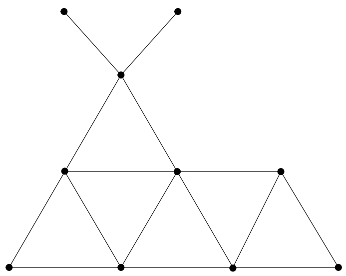
MT 5408 - GRAPH THEORY

Date: 29-04-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

SECTION A

Answer ALL the questions: **(10x2 = 20)**

1. Prove that every cubic graph has an even number of points.
2. Define a complete bipartite graph.
3. Give an example for an isomorphism between two graphs.
4. If $G_1 = K_2$ and $G_2 = C_4$ then find
   (i) G_1□G_2 (ii) $G_1 + G_2$
5. Define a tree with examples.
6. Prove that every Hamiltonian graph is 2-connected.
7. Define an eccentricity of a vertex *v* in a connected graph *G*.
8. Define the centre of a tree..
9. Is $K_{3,3}$ planar? If not justify your answer.
10. Find the chromatic number for the following graph *G*.



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SECTION B

Answer any FIVE questions: **(5x8 = 40)**

11. (a) Let *G* be a k-regular bigraph with bipartition (V_1, V_2) and $k > 0$. Prove that $|V_1| = |V_2|$.
    (b) Prove that $\delta \leq \frac{2q}{p} \leq \Delta$. **(5+3)**
12. (a) Prove that in any graph *G*, the number of points of odd degree is even.
    (b) Prove that any self-complementary graph has $4n$ or $4n+1$ vertices. **(4+4)**
13. If Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then prove that
    (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph.
    (ii) $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph.
14. (a) In a graph, prove that any $u - v$ walk contains a $u - v$ path.
    (b) Show that a closed walk of odd length contains a cycle. **(4+4)**
15. (a) If a graph *G* is not connected then prove that the graph $\bar{G}$ is connected.
    (b) Prove that a graph *G* with *p* points and $\delta \geq \frac{p-1}{2}$ is connected. **(4+4)**
16. If *G* is a graph with $p \geq 3$ vertices and $\delta \geq p/2$, then prove that *G* is Hamiltonian.
17. State and prove the five-colour theorem.
18. Prove that K_5 is non-planar.

SECTION C

Answer any TWO questions: **(2x20 = 40)**

19. (a) Prove that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.

    (b) Let G be a (p, q) graph then prove that $\Gamma(G) = \Gamma(\bar{G})$. **(15+5)**

20. (a) Prove that a graph G with at least two points is bipartite if and only if all its cycles are of even length.

    (b) Prove that every tree has a centre consisting of either one point or two adjacent points.

    (15+5)

21. (a) Prove that the following statements are equivalent for a connected graph G

    (i) G is eulerian.

    (ii) Every point of G has even degree.

    (iii) The set of edges of G can be partitioned into cycles.

    (b) If G is Hamiltonian, prove that for every non-empty proper subset S of V, the number of components of $G \setminus S$, namely, $\omega(G \setminus S) \leq |S|$. **(15+5)**

22. (a) Let G be a (p, q) graph then prove that the following statements are equivalent

    (i) G is a tree.

    (ii) Every two points of G are joined by a unique path.

    (iii) G is connected and $p = q + 1$.

    (iv) G is acyclic and $p = q + 1$.

    (b) Prove that the following statements are equivalent for any graph G:

    (i) G is 2-colourable.

    (ii) G is bipartite.

    (iii) Every cycle of G has even length. **(12+8)**

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