LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2016**

MT 6601 - APPLIED ALGEBRA

Date: 02-05-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Section A

Answer **ALL** questions: **10 × 2 = 20**

1. Show that the statement formula $(P \vee Q) \wedge \neg P$ is a tautology.
2. For the following truth table, form an equivalent statement formula involving the connectives *P* and *Q*

P	*Q*	?
T	*T*	*F*
T	*F*	*T*
F	*T*	*T*
F	*F*	*T*

3. Write an equivalent formula for $P \wedge (Q \rightleftarrows R)$ which contains neither biconditional nor conditional staement.
4. What are minterms of statement variables *P* and *Q*?
5. Show that the poset *P* = {(1, 2, 3, 4, 5) | *a* divides *b*} is not a lattice.
6. Prove that every chain is a distributive lattice.
7. Define a congruence relation.
8. Define a free semigroup on basis *B*.
9. Explain the term cascades of the automata $\mathscr{A}_1$ and $\mathscr{A}_2$.
10. Prove that the isomorphism of an automata is an equivalence relation on any set of automata.

Section B

Answer any **FIVE** questions: **5 × 8 = 40**

11. Prove that $(P \to Q) \vee R \Leftarrow (P \vee R) \to (Q \vee R)$.
12. Define the connective NAND and prove that it is not associative.
13. Show that if any two formulas are equivalent then their duals are also equivalent to each other.
14. Prove that the product of two lattices is a lattice.
15. Define a lattice $(L, \leq)$. In any lattice $(L, \leq)$ and $x, y, z \in L$, prove the following
    (i) $x \wedge (y \vee z) \geq (x \wedge y) \vee (x \wedge z)$ (ii) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$.
16. In a Boolean algebra *B*, prove that (i) $(x \wedge y)' = x' \vee y'$ (ii) $(x \vee y)' = x' \wedge y'$ for all *x*, *y* □ B.
17. State and prove Homomorphism theorem for semigroup.
18. Give the monoid of Parity check Automaton.

Section C

Answer any **TWO** questions: **2 × 20 = 40**

19. (a) Construct the truth table for $((P \wedge R) \wedge ((P \longrightarrow Q) \longrightarrow (R \longrightarrow S))) \longrightarrow S$.

(b) Obtain principal conjunctive normal form of the statement $(\neg P \longrightarrow R) \wedge (Q \rightleftarrows P)$. (12 +8)

20. Define the operations + and • on a Boolean algebra $B = (B, \wedge, \vee)$ by

$x + y = (x \wedge y') \vee (x' \wedge y)$, $x \bullet y = x \wedge y$, prove that $R(B) = (B, +, \bullet)$ is a Boolean ring with identity. Conversely, define the operation $\wedge$ and $\vee$ on a Boolean ring $R = (R, +, \bullet)$ with 1 by $x \vee y = x + y + xy$, $x \wedge y = x \bullet y$ and the complement $a' = a + 1$, then prove that Boolean algebra $B(R) = (R, \wedge, \vee)$.

21. Draw the graph of the automaton $\mathscr{A} = (Z, A, B, \delta, \lambda)$ where $A = B = Z = Z_4$, $\delta(z, a) = z + a$ and $\lambda(z, a) = za$.

22. Let $\mathscr{A}_1$ and $\mathscr{A}_2$ be Trigger Flip-Flop automaton. Construct the series composition $\mathscr{A}_1$ □ $\mathscr{A}_2$ of the automata $\mathscr{A}_1$ and $\mathscr{A}_2$.

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