LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2016**

MT 6603/MT 6600 – COMPLEX ANALYSIS

[UPTO 11-BATCH]

Date: 15-04-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer **ALL** questions: (10 x 2 = 20 marks)

1. Verify Cauchy-Riemann equations for the function: $f(z) = z^3$.
2. Show that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic.
3. Find the radius of convergence of the series $\frac{1}{2}z + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^2 + ...$
4. Using Cauchy's integral formula, evaluate $\frac{1}{2\pi i}\int_C \frac{z^2+2z+6}{z-1}$ where C is the circle $|z| = 3$.
5. Locate and classify the isolated singularities of $f(z) = \frac{e^z - 1}{z}$.
6. Define monomorphic function with an example.
7. Find the residue of $f(z) = \frac{ze^z}{(z-1)^3}$ at its poles.
8. State Argument theorem.
9. Define conformal mapping and give an example.
10. Find the fixed points of $f(z) = \frac{z+1}{z-1}$.

PART – B

Answer any **FIVE** questions: (5 x 8 = 40 marks)

11. Show that $|z_1 + z_2| \le |z_1| + |z_2|$ for any two complex numbers z_1, z_2. When is the inequality sharp?
12. Show that $u = e^x \cos y$ is harmonic and find its harmonic conjugate.
13. State and prove Liouvillie's theorem.
14. Compute $\int_{|z|=1} \frac{e^z}{z^3} dz$.
15. Find the Laurent series expansion of $f(z) = \frac{-1}{(z-1)(z-2)}$ valid in the regions (i) $1 < |z| < 2$ (ii) $0 < |z-1| < 1$.
16. Classify the singularities of the function $f(z) = \frac{z^2 - 5z + 6}{(z-2)(z+5)}$ and find the residue of $f(z)$ at $z = -5$.
17. State and prove residue theorem.
18. Find the bilinear transformation which maps the points z = -2, 0, 2 into the points w = 0, *i*, *-i* respectively.

PART – C

Answer any **TWO** questions (2 x 20 = 40 marks)

19. a) Derive Cauchy – Riemann equations for an analytic function $f(z)=u(x,y)+iv(x,y)$.

b) Prove that an analytic function with constant modulus must reduce to a constant. (10 + 10)

20. a) State and prove fundamental theorem of algebra.

b) State and prove Morera's theorem. (10 + 10)

21. a) State and prove Laurent's theorem.

b) Show that $\int_0^{\pi} \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}, (a>1)$. (10 + 10)

22. a) State and prove Rouche's theorem.

b) Prove that the totality of bilinear transformations which map $|z|=1$ onto $|w|=1$ must be of the form $w=k\frac{z-\alpha}{\bar{\alpha}z-1}$ where α is any complex number and $|k|=1$. (10+10)

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