LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2016**

MT 6608 – DISCRETE MATHEMATICS

Date: 21-04-2016

Time: 09:00-12:00

Dept. No.

Max. : 100 Marks

SECTION - A

ANSWER ALL QUESTIONS: (10 x 2 = 20 marks)

1) Construct the truth table for $P \vee \neg Q$.

2) Write the dual of $\neg(P \vee Q) \wedge \left(P \vee \neg(Q \wedge \neg S)\right)$.

3) Write down the min terms of P, Q and R.

4) Obtain the principle disjunctive normal forms of (i) $P \to Q$ (ii) $P \vee Q$.

5) Define semi group.

6) Define sub monoid and give an example.

7) Define Lattice.

8) Define partially ordered relation.

9) Define Boolean Algebra.

10) Draw the diagram for the Boolean Algebra $\langle \rho(S), \cap, \cup \rangle$ where $S = \{a, b\}$.

SECTION - B

ANSWER ANY FIVE QUESTIONS: (5 x 8 = 40 marks)

11) Construct the truth table for $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$.

12) Show that $\left((P \vee Q) \wedge \neg\left(\neg P \wedge (\neg Q \vee \neg R)\right)\right) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

13) Obtain the principle disjunctive normal form of $\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$.

14) Write the following sentences in the symbolic form:
  (i) If there is a flood then the crop will be destroyed.
  (ii) If either Jerry takes Calculus or Ken takes Sociology, then Larry will take English.

15) Prove that for any commutative monoid $(M, *)$, the set of all idempotent elements of M forms a submonoid.

16) Define (i) Lattice homomorphism
   (ii) Lattice endomorphism
   (iii) Lattice automorphism.

17) Let $\langle L, \leq \rangle$ be a Lattice. Then prove that for any $a, b, c \in L$, the inequality

   $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ holds.

18) Obtain the values of the Boolean forms (i) $x_1 * \left(x_1' \oplus x_2\right)$ (ii) $x_1 * x_2$ (iii) $x_1 \oplus (x_1 * x_2)$.

SECTION - C

ANSWER ANY TWO QUESTIONS: (2 x 20 = 40 marks)

19) (a) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$.

   (b) Obtain the p.d.n.f. of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.   **(10+10)**

20) (a) Show that $\langle Z_6, +_6 \rangle$ is a semi group. Also find all the sub semi groups of $\langle Z_6, +_6 \rangle$.

(b) Prove that the composition of semigroup homomorphisms is also a semigroup
homomorphism. **(10+10)**

21) (a) State and prove the four properties of Lattices.
(b) Show that in a lattice if $a \leq b \leq c,$ then $a \oplus b = b * c.$ **(16+4)**

22) (a) Write down the following Boolean expressions in an equivalent sum - of – products canonical form in
three variables x_1, x_2 and x_3 : (i) $x_1 * x_2$ (ii) $x_1 \oplus x_2$ (iii) $\left(x_1 \oplus x_2 \right)' * x_3.$
(b) Define the following: (i) complete lattice (ii) bounded lattice (iii) complemented lattice
(iv) distributive lattice. **(10+10)**

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