LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – APRIL 2017

16PMT1MC05- PROBABILITY THEORY AND STOCHASTIC PROCESS

Date: 05-05-2017 Dept. No. Max. : 100 Marks
TIME 09:00-12:00

Answer ALL Questions:

1. (a) A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k, (ii) Evaluate $P(X<6), P(X\geq 6)$, and $P(0<X<5)$ (iii) If $P(X\leq a)>\frac{1}{2}$, find the minimum value of a.

OR

(b) Derive the Chi- square distribution. (5)

(c) Let X be a continuous random variable with p.d.f given by

$$F(x)=\begin{cases} kx, & 0\leq x<1 \\ k, & 1\leq x<2 \\ -kx+3x, & 2\leq x<3 \\ 0, & elsewhere \end{cases}$$

(i)Determine the constant k, (ii) Determine $F(x)$, the c.d.f and (iii) If x_1, x_2, x_3 are three independent observation from X, what is the probability that exactly one of these three numbers is larger than 1.5? (8)

(d) A random sample of 10 boys had the following I.Q's : 70, 120, 110, 101, 85, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100? Find the reasonable range in which most of the mean I.Q values of sample of 10 boys lie. (7)

OR

(e) Derive the constants of F- distribution.

(f) Show that if v is even, $P=\frac{1}{2^{(v-2)/2}\Gamma(v/2)}\int_{\chi}^{\infty} exp(-\chi^2/2)\chi^{v-1}d\chi$. (5 + 10)

2. (a) Explain the generalized form of Bienayme- Chebyshev's inequality.

OR

(b) Does there exist a variate X for which $P[\mu_x-2\sigma\leq X\leq\mu_x+2\sigma]=0.6$? (5)

(c) State and prove Chebyshev's inequality.

(d) Use chebyshev's inequality to determine how many times a fair coin must be tossed in order that the probability will be at least 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6. (10 + 5)

OR

(e) State and prove Khinchin's theorem. (15)

3. (a) Let $x_1, x_2, \dots, x_n$ be a random sample from a uniform population on $[0,\theta]$. Find a sufficient estimator for θ.

OR

(b) If T_1 and T_2 are two unbiased estimators of $\gamma(\theta)$, having the same variance and ρ is the correlation between them, then show that $\rho\geq 2e-1$, where e is the efficiency of each estimator. (5)

(c) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ:

(i) $t_1 = \frac{X_1+X_2+X_3+X_4+X_5}{5}$, (ii) $t_2 = \frac{X_1+X_2}{2} + X_3$, (iii) $t_3 = \frac{2X_1+X_2+\lambda X_3}{3}$

where λ is such that t_3 is an unbiased estimator of μ. Find λ. Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3. (8)

(d) A minimum variance unbiased estimator is unique in the sense that if T_1 and T_2 are M. V. U estimators for $\gamma(\theta)$ then prove that $T_1 = T_2$, almost surely. (7)

OR

(e) Find the M. L. E of the parameters α and λ (λ being large) of the distribution $f(x;\alpha,\lambda) = \frac{1}{\Gamma(\lambda)}\left(\frac{\lambda}{\alpha}\right)^{\lambda} e^{\frac{-\lambda x}{\alpha}} x^{\lambda-1}, 0 \le x \le \infty, \lambda > 0$, where $\frac{\partial}{\partial\lambda}\log\Gamma(\lambda) = \log\lambda - \frac{1}{2\lambda}$ and $\frac{\partial^2}{\partial\lambda^2}\log\Gamma(\lambda) = \frac{1}{\lambda} + \frac{1}{2\lambda^2}$.

(8)

(f) For the double Poisson distribution: $p(x) = P(X = x) = \frac{1}{2}\cdot\frac{e^{-m_1}{m_1}^x}{x!} + \frac{1}{2}\cdot\frac{e^{-m_2}{m_2}^x}{x!}$; $x = 0,1,2,\ldots$. Show that the estimates for m_1 and m_2 by the method of moments are ${\mu_1}' \pm \sqrt{{\mu_2}' - {\mu_1}' - {\mu_1}'^2}$. (7)

4. (a) Derive a relation between two successive ordered observations of a p. d. f. with continuous random samples $Z_1, Z_2, \ldots, Z_n$.

OR

(b) If $x \ge 1$, is the critical region for testing $H_0: \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population $f(x,\theta) = \theta e^{-\theta x}, 0 \le x < \infty$, obtain the values of type I and type II errors. (5)

(c) Prove that most powerful (MP) or uniformly (UMP) critical region (CR) is necessarily unbiased.

(i) if W be an Most Powerful Critical Region of size α for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$, when it is necessarily unbiased.

(ii) Similarly if W be Uniformly Most Powerful Critical Region of size α for testing $H_0: \theta = \theta_0$ against $H_1: \theta \in \Theta$, then it is also unbiased.

(8)

(d) Write a brief note on Wald-Wolfowitz run test. (7)

OR

(e) Given the frequency function $f(x,\theta) = \begin{cases} \frac{1}{\theta} & 0 < x < \infty, \theta > 0 \\ 0 & elsewhere \end{cases}$ and what you are testing the null hypothesis $H_0: \theta = 1$ against $H_1: \theta = 2$, by means of a single observed value of x. What would be the sizes of the type I and type II errors, if you choose the interval (i) $0.5 \le x$, (ii) $1 \le x \le 1.5$ as the critical region? Also obtain the power function of the test.

(f) Write down the advantages and disadvantages of non-parametric tests. (8 + 7)

5. (a) Write a short note on classification of stochastic process.

OR

(b) Describe the procedure used in median test. (5)

(c) Prove that a homogeneous Markov chain $\{X_n\}$ satisfies the relation $P_{ij}^{(n+m)} = \sum_k P_{ik}^{(n)} P_{kj}^{(m)}$. (8)

(d) Let $X \sim N(\mu, 4)$, μ unknown. To test $H_0: \mu = -1$ against $H_1: \mu = 1$, based on a sample of size 10 from this population, we use the critical region: $x_1 + 2x_2 + \cdots + 10x_{10} \ge 0$. What is its size? What is the power of the test? (7)

OR

(e) Briefly explain a time dependent general birth and death process in stochastic process. (20)

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