



# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

## B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – APRIL 2017

### 16UMT1MC01- ALGEBRA AND CALCULUS - I

Date: 19-04-2017  
09:00-12:00

Dept. No.

Max. : 100 Marks

#### PART – A

Answer ALL questions

(10 X 2 = 20)

1. Find the  $n^{\text{th}}$  derivative of  $y = \sin ax$ .
2. Find the subnormal of the parabola  $y^2 = 4ax$ .
3. State the conditions for maxima and minima of two variables.
4. Write the steps used in Lagrange's method of undetermined multipliers.
5. Write the  $p-r$  equation of a curve.
6. Define an asymptote to a curve.
7. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 - 9x^2 + 108 = 0$ , Find the value of  $\sum \alpha^2$
8. Form the quadratic equation one of whose roots is  $\sqrt{5} - 1$ .
9. How many real root are there in the equation  $x^5 - 6x^2 - 4x + 5 = 0$ ?
10. What is the condition for the roots of equation  $x^3 + px^2 + qx + r = 0$  to be in harmonic progression.

#### PART – B

Answer any FIVE questions

(5 X 8 = 40)

11. If  $y = \sin^{-1} x$ , Prove that  $(1-x^2)y_2 - xy_1 = 0$  and  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ .
12. Find the slope of the tangent with the initial line for the cardioid  
$$r = a(1 - \cos \theta) \text{ at } \theta = \frac{\pi}{6}$$
13. Find the minimum values of  $2(x^2 - y^2) - x^4 + y^4$ .
14. Find the radius of curvature at 't' on the curve  $x = 6t^2 - 3t^4, y = 8t^3$ .
15. Find the asymptotes of the curve  $x^3 + 3x^2y - xy^2 - 3y^3 + x^2 - 2xy + 3y^2 + 4x + 7$ .
16. Solve the equation  $x^3 - 19x^2 + 114x - 216 = 0$ , given that the roots are in GP.
17. Find  $\frac{1}{\alpha^5} + \frac{1}{\beta^5} + \frac{1}{\gamma^5}$  where  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + 2x^2 - 3x - 1 = 0$ .
18. Solve the equation  $x^3 - 6x - 9 = 0$  using Cardon's method.

**PART- C**

**ANSWER Any TWO Questions**

**(2 X 20 = 40)**

19. a) If  $y = (x + \sqrt{1+x^2})^m$ , Prove that  $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$ .

b) Using Lagrange's multipliers method find the minimum value of u if

$$u = a^3x^2 + b^3y^2 + c^3z^2 \text{ where } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 \quad \textbf{(8 + 12)}$$

20. a) Find the radius of curvature of the cardioid  $r = a(1 - \cos \theta)$

b) Obtain the evolute of the curve  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$

**(8 + 12)**

21. a) Solve  $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$ .

b) If the sum of the two roots of the equation  $x^4 + px^3 + qx^2 + rx + s = 0$  equals

the sum of the other two, prove that  $p^3 + 8r = 4pq$  **(10 + 10)**

22. Use Horner's method to calculate the positive root of the equation

$$x^3 + 24x - 50 = 0 \text{ correct to two places of decimals.} \quad \textbf{(20)}$$

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