LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – APRIL 2017

16UMT1MC02- ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 21-04-2017   Dept. No.   Max. : 100 Marks
09:00-12:00

PART – A

Answer all the questions   **$10 \times 2 = 20$**

1. Write down the expansion for $cos^5\theta$ in terms of θ.

2. If $\frac{sin\theta}{\theta} = \frac{5045}{5046}$, show that $\theta = 1°58'$ approximately .

3. Prove that $\cos h^2 x + sinh^2 x = cosh2x$.

4. Find $Log(1 - i)$

5. Find the eigen values of $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

6. Define the characteristics equation and characteristic values of the matrix A.

7. Write down the condition for the lines $lx + my + n = 0$ and $l_1x + m_1y + n_1 = 0$ to be conjugate to each other.

8. Define conjugate diameters of the ellipse.

9. Find the asymptotes of the hyperbola
$$3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$$

10. Write down the polar equation of a straight line.

PART – B

Answer any FIVE questions   **$5 \times 8 = 40$**

11. Express $cos8\theta$ in terms of $sin\theta$.

12. Find $\lim\limits_{\theta \to 0} \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$

13. If $cos\alpha. cosh\beta = cos\varphi, sin\alpha \, sinh\beta = sin\varphi,$ prove that

$sin\varphi = \pm sin^2\alpha = \pm sinh^2\beta$.

14. Deduce the expansion of $tan^{-1}x$ in powers of x from the expansion of $\log(a + ib)$.

15. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$.

16. Calculate A^4 when $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

17. Find the locus of the poles of all tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$.

18. If e, e_1 are the eccentricities of a hyperbola and its conjugate, show that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$

PART – C

Answer any TWO questions $2 \times 20 = 40$

19. a) Prove that $\frac{sin6\theta}{sin\theta} = 32cos^5\theta - 32cos^3\theta + 6cos\theta$

   b) Expand $sin^3\theta cos^5\theta$ in a series of sines of multiples of θ **(10+10)**

20. a) If $\cos(x + iy) = cos\theta + isin\theta$, prove that $cos2x + cosh2y = 2$

   b) Separate into real and imaginary parts $tan^{-1}(x + iy)$. **(10+10)**

21. Diagonalise the matrix $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.

22. a) A tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose centre is C meets the circle $x^2 + y^2 = a^2 + b^2$ at

Q and Q'. Prove that CQ and CQ' are conjugate diameters of the ellipse.

b) Trace the curve $\frac{10}{r} = 3cos\theta + 4sin\theta + 5$ **(10+10)**

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