LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

SECOND SEMESTER – APRIL 2017

16UMT2AL01- MATHEMATICS FOR PHYSICS - II

Date: 27-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

SECTION A

Answer **ALL** the questions: **(10x2 = 20)**

1. Evaluate $\int (ax^2 + bx + c)\,dx$.
2. Find the value of $\int_0^{\pi/2} \sin^6 x\,dx$.
3. State any two properties of definite integral.
4. Prove that $\beta(m,n) = \beta(n,m)$.
5. Solve $(1-x^2)\frac{dy}{dx} + xy = 5x$.
6. Write the criterion for $Mdx + Ndy = 0$ to be exact.
7. Evaluate $\int_0^a \int_0^b xy(x-y)\,dydx$.
8. Find $\frac{\partial(x,y)}{\partial(r,\theta)}$ when $x = r\cos\theta$ and $y = r\sin\theta$.
9. Prove that $\nabla . r = 3$ and $\nabla \times r = 0$.
10. State Gauss Divergence Theorem.

SECTION B

Answer any **FIVE** questions: **(5x8 = 40)**

11. Evaluate $\int \frac{2dx}{(1-x)(1+x^2)}$.
12. Establish a reduction formula for $I_n = \int \tan^n x\,dx$; hence find $\int_0^{\pi/4} \tan^3 x\,dx$.
13. Prove that $\int_0^{\pi} \theta \sin^3\theta\,d\theta = \frac{2\pi}{3}$.
14. Solve $(D^2 - 4D + 3)y = e^{-x}\sin x$.
15. Solve $x\frac{dy}{dx} + y\log x = e^x x^{1-\frac{1}{2}\log x}$.
16. Evaluate $\iint r\sqrt{a^2 - r^2}\,drd\theta$ over the upper half of the circle $r = a\cos\theta$.
17. Compute the divergence and curl of the vector $F = xyzi + 3x^2yj + (xz^2 - y^2z)k$ at $(1,2,-1)$.
18. Prove that $\int_C \phi dr = \iint_S n \times \nabla\phi\,dS$.

SECTION C

Answer any **TWO** questions: **(2x20 = 40)**

19. (a) Derive the reduction formula for $I_n = \int \sin^n x dx$.

(b)Solve $\left(3D^2 + D - 14\right) y = 13e^{2x}$.

(10+10)

20. (a) Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(b) Evaluate $\int_0^1 x^7 (1-x)^8 dx$ using Beta & Gamma function. **(15+5)**

21. (a) Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xydxdy$ and evaluate it.

(b) Evaluate $\iint \left(x^2 + y^2\right) dxdy$ over the region for which x, y are each ≥ 0 and $x + y \leq 1$.

(14+6)

22. (a) Evaluate $\int \dfrac{2x+3}{x^2+x+1} dx$.

(b) Find by Green's Theorem the value of $\int_C \left(x^2 ydx + ydy\right)$ along the closed curve C formed by $y^2 = x$ and $y = x$ between $(0,0)$ and $(1,1)$.

(10+10)

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