LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2017

16UMT2MC01- ALGEBRA AND CALCULUS - II

Date: 22-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART – A
Answer ALL questions: (10 X 2 = 20)

1. Evaluate $\int_{0}^{\pi/6} \cos^2 \frac{x}{2} dx$.
2. If f is an even function, show that $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.
3. Evaluate $\int_{0}^{3}\int_{0}^{2} (x+y)dxdy$.
4. If $x = r\cos\theta$ and $x = r\sin\theta$, find the Jacobian of x and y with respect to r and θ
5. Define Beta and Gamma functions.
6. Using Gamma functions evaluate $\int_{0}^{1} x^7(1-x)^8 dx$.
7. Show that $\left\{\frac{n}{n+1}\right\}$ is a monotonic increasing sequence..
8. State Cauchy's root test for convergence.
9. Write the expansion of e and e^{-1} by including their n^{th} terms.
10. Expand $(1+x)^{-p/q}$.

PART – B
Answer any FIVE questions:(5 X8 = 40)

11. Show that $\int_{0}^{\pi} \frac{x\tan x}{\sec x + \tan x} dx = \frac{\pi(\pi-2)}{2}$.
12. Find the length of one loop of the curve $3ay^2 = x(x-a)^2$.
13. Evaluate $\iint xy\,dx\,dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$
14. Prove that $\beta(m,n) = \int_{0}^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}} dy$
15. Evaluate using Gamma functions: $\int_{0}^{\infty} e^{-x^2} dx$
16. Use D'Alembert's ratio test to test the convergence of $\sum_{n=0}^{\infty} \frac{n^3+1}{2^n+1}$.

17. Find the greatest term in the expansion of $(3x+2)^{35}$ when $x=2$.

18. Find the sum to infinity of the series $1+\frac{3}{4}+\frac{3\cdot 5}{4\cdot 8}+\frac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12}+\cdots$.

PART– C

Answer any TWO questions: (2 X 20 = 40)

19. a) Evaluate $\int_0^{\pi/2} \log \sin x \, dx$

b) Find the reduction formula for $I_n = \int \sin^n x \, dx$ and hence find $\int_0^{\pi/2} \sin^6 x \, dx$ and $\int_0^{\pi/2} \sin^5 x \, dx$.

(10 + 10)

20. a) Change the order of integration in the integral $\int_0^{a}\int_{x^2/a}^{2a-x} xy \, dx \, dy$ and evaluate.

b) Evaluate $\iint_R (x-y)^4 e^{x+y} \, dx \, dy$, where R is the square with vertices $(1,0),(2,1),(1,2)$ and $(0,1)$.

(10 + 10)

21. a)Prove that $\beta(m,n)=\frac{\Gamma m \Gamma n}{\Gamma(m+n)}$

b) Prove that $\beta(m,n)=2\int_0^{\pi/2} Sin^{2m-1}\theta Cos^{2n-1}\theta d\theta.$

(15 + 5)

22. a) Find the sum to infinity of the series $1+\frac{1+3}{2!}+\frac{1+3+3^2}{3!}+\frac{1+3+3^2+3^3}{4!}+\cdots$

b) Test the convergence of $1+\frac{2}{2!}x+\frac{3^2}{3!}x^2+\frac{4^3}{4!}x^3+\frac{5^4}{5!}x^4+\ldots$.

(10 + 10)

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