LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2017

16UMT2MC02- ANA. GEO. OF 3D, FOURIER SERIES AND NUMBER THEORY

Date: 25-04-2017   Dept. No.   Max. : 100 Marks
01:00-04:00

PART – A

ANSWER ALL THE QUESTIONS   (10 x 2 = 20)

1. Find the equation to the plane through (1,2,3) parallel to the plane 4x+5y-3z+7 = 0.

2. Write the equation of the symmetric form of the straight line.

3. Find the equation of the sphere whose centre is (1,2,3) and radius is 4.

4. Write the equation of the tangent plane to the sphere.

5. Write the formula for Fourier series.

6. Define an odd function with an example.

7. Find the number of divisors of 360.

8. State Fermat's theorem.

9. Show that $(b + c - a)^2 + (c + a - b)^2 + (a + b - c)^2 \geq bc + ca + ab.$.

10. Show that $n^n > 1 . 3 . 5 . \ldots (2n-1)$.

PART – B

ANSWER ANY FIVE QUESTIONS   (5 x 8 = 40).

11. Show that, if a plane has intercepts a, b, c on the coordinate axes and is at a distance p from the origin

    , then $\dfrac{1}{a^2} + \dfrac{1}{b^2} + \dfrac{1}{c^2} = \dfrac{1}{p^2}$.

12. Show that the lines $\dfrac{x-2}{2} = \dfrac{y-3}{-1} = \dfrac{z+4}{3}$; $\dfrac{x-3}{1} = \dfrac{y+1}{3} = \dfrac{z-1}{1}$ are coplanar. Find their point of

    intersection and the equation to the plane containing them.

13. Find the equation of the sphere whose centre is the point (6,-1,2) and which touches the plane
    $2x - y + 2z - 2 = 0$.

14. Find the equation of the sphere through the points (0,-2,3), (1, 5, -1), (2, 0, 1) and (4, -1, 2).

15. Find a sine series for f(x) = c in the range 0 to π.

16. Find the highest power of 3 dividing 1000!.

17. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.

18. Show that $(x^m + y^m)^n < (x^n + y^n)^m$ if m > n.

PART – C

ANSWER ANY TWO QUESTIONS   **(2x 20 = 40)**

19. (a) Find the equation of the plane through $(2, -1, 1)$ and perpendicular to the line joining the points

$(3,4,-1)$ and $(2,-1,5)$.

(b) Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \quad and \quad \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$

20. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the equation of the circumcircle of the triangle ABC and determine also the coordinates of the centre and radius.

21. (a) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \le x \le \pi)$.

(b) Show that 8th power of any number is of the form 17m or $17m \pm 1$.

22. (a) State and prove Wilson's theorem.

(b) If $s = a_1 + a_2 + ... + a_n$ then show that $\frac{s}{s-a_1} + \frac{s}{s-a_2} + ... + \frac{s}{s-a_n} > \frac{n^2}{n-1}$ unless

$a_1 = a_2 = ... = a_n$.

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