LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRSTSEMESTER – APRIL 2017

M 102 – GENERAL MATHEMATICS - 1

Date: 29-04-2017   Dept. No.   Max. : 100 Marks
09:00-12:00

Answer any **SIX** questions

1. (a) Find the nth derivative of $x^2 log x$.

   (b) If $y = \left(x + \sqrt{1+x^2}\right)^m$ show that $(1+x^2)y_2 + xy_1 = m^2 y$. Hence prove that
   $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$.   **(8+9)**

2. (a) Find the lengths of the subtangent, subnormal, tangent and normal at (a,a) on the cissoid $y^2 = \frac{x^3}{2a-x}$.

   (b) Find the angle of intersection of the cardioids $r = a(1+cos\theta)$ and $r = b(1-cos\theta)$.
                                                                 (9+8)

3. (a) If $u = x^3 + y^3 + z^3 + 3xyz$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$.
   (b) Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point $(1,1)$.   **(9+8)**

4. (a) Evaluate $\int \frac{3x+7}{2x^2+3x-2} dx$.

   (b) Integrate $x^2 e^{3x}$ with respect to x.   **(9+8)**

5. (a) Prove that $\int_0^{\pi/2} \frac{\sqrt{sin\,x}}{\sqrt{sin\,x}+\sqrt{cos\,x}} dx = \frac{\pi}{4}$.

   (b) Evaluate $\int_0^{\pi/2} sin^8 x\, cos^6 x\, dx$.   **(9+8)**

6. (a) Derive the standard equation of parabola.
   (b) Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.   **(8+9)**

7. (a) Solve the equation $x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$ which has a root $2 + \sqrt{3}$.

   (b) Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2 and find the transformed equation.
                                                                 (8+9)

8. (a) Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in A.P.

   (b) Solve the reciprocal equation $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$.
                                                                 (8+9)

9. (a) Express $cos\,6\theta$ in terms of $cos\,\theta$.

   (b) Prove that $sin^4\theta cos^2\theta = \frac{1}{2^5}(cos\,6\theta - 2cos\,4\theta - cos\,2\theta + 2)$.   **(9+8)**

10. (a) Separate into real and imaginary parts of (i) $sinh(x+iy)$ (ii) $cosh(x+iy)$.

    (b) If $sin(A+iB) = x + iy$, prove that (i) $\frac{x^2}{cosh^2 B} + \frac{y^2}{sinh^2 B} = 1$ (ii) $\frac{x^2}{sin^2 A} - \frac{y^2}{cos^2 A} = 1$.   **(9+8)**

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