LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – APRIL 2017

MT 1101- MATHEMATICS FOR STATISTICS

Date: 24-04-2017   Dept. No.   Max. : 100 Marks
01:00-04:00

PART A

Answer all the questions: (10 X 2 = 20)

1. If $f(x) = (4x^2 - 1)(3x - 5)$, find the values of $f(2)$ and $f\left(\frac{1}{2}\right)$.

2. Differentiate $\frac{x^3}{3x-2}$ with respect to x.

3. For what values of x is the curve $y = 3x^2 - 2x^3$ concave upwards?

4. Find the points of inflexion on $y = x^3 - 9x^2 + 7x - 6$.

5. Using Maclaurin's series, expand $\cos x$ as an infinite series.

6. Find the first order partial differential coefficients of $u = cos(7x + 4y)$.

7. Integrate $x^2 e^x$ with respect to x.

8. Evaluate $\int \frac{3x^2 dx}{4+x^3}$.

9. Solve $\int_0^a \int_0^b xy \, dy \, dx$.

10. Find $\int_1^2 (2x^3 + x - 4) \, dx$.

PART B

Answer any FIVE questions: (5 X 8 = 40)

11. (a) If $y = log\left[tan(e^{x^2})\right]$, find $\frac{dy}{dx}$.

(b) Prove that the tangents to the curve $y = x^2 - 5x + 6$ at the points $(2,0)$ and $(3,0)$ cut at

right angles.   (4+4)

12. Find the points of inflexion on the curve $y = \frac{a^2 x}{x^2 + a^2}$.

13. Using mean value theorem, determine c, lying between a and b, when
    (i) $f(x) = x^3 - 2x^2, a = 2, b = 5$
    (ii) $f(x) = x^3 + x, \quad a = 1, b = 2$.

14. Prove that $log(1 + x + x^2) = x + \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{4}x^4 + \cdots$

15. Integrate $sin2x cos3x$ with respect to x.

16. Evaluate $\int \frac{x}{x^2+x+1} \, dx$.

17. Prove that $\int_0^{\frac{\pi}{2}} log \, sinx \, dx = \frac{\pi}{2} log\left(\frac{1}{2}\right)$.

18. Evaluate $\iint (x^2 + y^2) \, dxdy$ over the region for which $x, y \geq 0$ and $x + y \leq 1$.

PART C

Answer any TWO questions: (2 X 20 = 40)

19. (a) If $f(x) = x^3 + x^2 + x - 1$, simplify $f(x + 1) - 3f(x) + 2f(x - 1)$

(b) If $y = cosx\,cos2x\,cos3x$, find $\frac{dy}{dx}$.

(c) Differentiate $x^{(logx)^2}$ with respect to $(xlogx)(loglogx)$ (7+6+7)

20. (a) If $u = \frac{1}{r}$ and $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

   (b) Find the minimum values of $x^2 + y^2 + z^2$ if $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.   (10+10)

21. (a) Verify Euler's theorem when $u = x^3 - 3x^2y + 3xy^2 + y^3$.
   (b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}\,dx = \frac{\pi}{4}$.   (10+10)

22. (a) By transforming into polar coordinates, evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)dxdy$.
   (b) Evaluate $\int \frac{5x+3\,dx}{x^2+4x+10}$.   (10+10)

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