



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – APRIL 2017

MT 1502- ALGEBRA AND CALCULUS - I

Date: 18-04-2017
01:00-04:00

Dept. No.

Max. : 100 Marks

SECTION – A

(Answer ALL questions)

(10 × 2 = 20)

1. Find the n^{th} derivative of $e^x \sin x$.
2. Find the slope of the curve $r = a(1 - \cos \theta)$ at $\theta = \pi/2$.
3. Write the conditions for the maxima and minima of functions of two variables.
4. Write the steps used in Lagrange's method of undetermined multipliers.
5. What is the asymptote of a given curve?
6. Find the coordinates of the centre of curvature of the curve $y = x^2$ at the point $(1/2, 1/4)$.
7. Determine the quadratic equation having $\sqrt{2} + 1$ as a root.
8. Define reciprocal equation.
9. Define Descartes rule of signs for negative roots.
10. Show that the equation $x^3 - 6x - 13 = 0$ has one real root between 3 and 4.

SECTION – B

(Answer any FIVE questions)

(5 × 8 = 40)

11. If $y = \cos(m \cos^{-1} x)$, show that $(1-x^2)y_{n-2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$.
12. Find the angle between the radius vector and the tangent for the curve $r^2 = a^2 \cos 2\theta$ at $\theta = \frac{\pi}{6}$.
13. Investigate the maximum and the minimum value of $4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.
14. Find the asymptotes of the cubic $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.
15. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin \theta)$ and $y = a(1 - \cos \theta)$ is $4a \cos \frac{\theta}{2}$.
16. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.
17. Find the sum of the cubes of the roots of the equation $x^5 = x^2 + x + 1$.
18. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$.

SECTION – C

(Answer any TWO questions)

(2 × 20 =40)

19(a) If $y = e^{m \sin^{-1} x}$, prove that $(1-x^2) y_{n+2} - (2n+1)xy_{n+1} - (m^2 + n^2) y_n = 0$. **(10+10)**

(b) Find the maxima and minima of the function $f(x,y) = 12xy - 3y^2 - x^2$ subject to $x+y=16$.

20(a) Show that the evolute of the cycloid $x=a(\theta - \sin\theta)$; $y=a(1 - \cos\theta)$ is another cycloid.

(b) Find the asymptotes of $x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y = 1$. **(10+10)**

21(a) Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetic progression if $2p^3 - 9pq + 27r = 0$. Hence solve $x^3 - 6x^2 + 13x - 10 = 0$.

(b) Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$. **(10+10)**

22 (a) Solve the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$ by finding the rational roots.

(b) Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to 3 places of decimal. **(10+10)**
