LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – APRIL 2017

MT 1503- ANALYTICAL GEOMETRY OF 2D, TRIG. & MATRICES

Date: 19-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART – A

ANSWER ALL THE QUESTIONS: **(10 × 2 = 20 marks)**

1. Write the expansion of $\tan n\theta$ in powers of $\tan\theta$.
2. Expand $\sin^4\theta$ in a series of cosines of multiples of θ.
3. Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$.
4. Find $\log(1-i)$.
5. Define (a) Singular matrix and (b) Skew symmetric matrix.
6. Find A^3 when $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$.
7. Show that the perpendicular tangents to a parabola intersect on the directrix.
8. State any two properties of conjugate diameters.
9. Define rectangular hyperbola.
10. Find the distance between the points (r_1, θ_1) and (r_2, θ_2).

PART - B

ANSWER ANY FIVE QUESTIONS: **(5 × 8 = 40 marks)**

11. Expand $\cos^6\theta$ in series of cosines of multiples of θ.
12. Evaluate $\lim_{x\to\frac{\pi}{2}} \left(\frac{\sin x + \cos 2x}{\cos^2 x}\right)$
13. If $\cos(x+iy) = \cos\theta + i\sin\theta$, then prove that $\cos 2x + \cosh 2y = 2$.
14. If $\log\sin(\theta + i\phi) = A + iB$, then prove that $2e^{2A} = \cosh 2\phi - \cos 2\theta$.
15. Verify that the following matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation.
16. The polar of a point P with respect to the parabola $y^2 = 4ax$ meets the curve in Q and R. Show that if P lies on the line $lx + my + n = 0$, then the middle point of QR lies on the parabola $l(y^2 - 4ax) + 2a(lx + my + n) = 0$.
17. (a) Find the locus of the middle points of a series of parallel chords of an ellipse. **(5+3)**
    (b) When will the tangents at the extremities of a chord intersect on the diameter bisecting the chord.
18. Trace the curve $\frac{12}{r} = 4 + \sqrt{3}\cos\theta + \sin\theta$

PART – C

ANSWER ANY TWO QUESTIONS: **$(2 \times 20 = 40$ marks)**

19. (a)Prove that $\frac{\sin 6\theta}{\sin \theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$ **(10)**

(b)Prove that $\cos^5\theta\sin^3\theta = \frac{-1}{128}(\sin 8\theta + 2\sin 6\theta - 2\sin 4\theta - 6\sin 2\theta)$ **(10)**

20. (a) If $\cos\alpha\cosh\beta = \cos\emptyset$, $\sin\alpha\sinh\beta = \sin\emptyset$, then prove that $\sin\emptyset = \pm\sin^2\alpha = \pm\sinh^2\beta$ **(10)**

(b) Separate into real and imaginary parts $\tan^{-1}(x + iy)$ **(10)**

21. Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ **(20)**

22. (a) Show that the locus of the intersection of tangents to $y^2 = 4ax$ which intercept a constant length d on the directrix is $(y^2 - 4ax)(x + a)^2 = d^2x^2$. **(12)**

(b) The asymptotes of a hyperbola are parallel to $2x + 3y = 0$ and $3x - 2y = 0$. Its centre is at (1,2) and it passes through the point (5,3). Find its equation and its conjugate. **(08)**

****