LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – APRIL 2017

MT 1818- DIFFERENTIAL GEOMETRY

Date: 04-05-2017 Dept. No. Max. : 100 Marks
09:00-12:00

Answer ALL the Questions:

1. a) Find the length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $x = cosh\left(\frac{z}{c}\right)$. (5)

OR

b) For the curve $x = a(3u - u^3)$, $y = 3au^2$, $z = a(3u + u^3)$, show that the curvature and torsion are equal. (5)

c) Find the directions and equations of the tangent, normal and binormal and also obtain the normal, rectifying and osculating planes at a point on the circular helix $\vec{x} = \left(acos\left(\frac{s}{c}\right), asin\left(\frac{s}{c}\right), \frac{bs}{c}\right)$. (15)

OR

d) State and prove Serret-Frenet formula and express $\bar{t}, \bar{n}$ and $\bar{b}$ in terms of Darboux vector. (15)

2. a) Find the plane that has three point contact at the origin with the curve $x = u^4 - 1, y = u^3 - 1, z = u^2 - 1$. (5)

OR

b) Find the necessary and sufficient condition that a curve to be a helix. (5)

c) Find the curve whose intrinsic equations are $\kappa = \frac{1}{\sqrt{2as}}$ and $\tau = 0$. (15)

OR

d) Derive the equation of involute of a curve. Also find the curvature and torsion of an involute. (15)

3. a) Prove that the first fundament form of a surface is a positive definite quadratic form in *du*, *dv*. (5)

OR

b) Define the envelope of the system of surface. Find the envelope of the sphere $(x - acos\theta)^2 + (y - asin\theta)^2 + z^2 = b^2$. (5)

c) Derive polar developable and rectifying developable associated with a space curve. (15)

OR

d) Find the equation of the developable surface whose generating lines pass through the curves $y^2 = 4ax$, $z = 0$; $x^2 = 4ay$, $z = c$. Also find the edge of regression. (15)

4. a) State and prove Euler's theorem. (5)

OR

b) Prove that the curves of the family $\frac{v^3}{u^2} = a\ constant$ are geodesic on a surface with metric $v^2du^2 - 2uvdudv + 2u^2dv^2$ ($u > 0$, $v > 0$). (5)

c) Define Dupin Indicatrix. Derive equation of Dupin Indicatrix. (15)

OR

d) (i) Find an equation giving principal curvatures.
(ii) Find the principal radii and lines of curvature of the surface $y\cos\left(\frac{z}{a}\right) = x\sin\left(\frac{z}{a}\right)$. (8 + 7)

5. a) State and prove Hilbert's Theorem. (5)

OR

b) Derive Rodriquez's formula for the lines of curvature from Weingarten's equation. (5)

c) Derive partial differential equation of surface theory. (15)

OR

d) Illustrate the fundamental theorem of surface theory for the differential forms
I: $du^2 + \cos^2 u dv^2$; II: $du^2 + \cos^2 u dv^2$
Using the equations of Gauss and Weingarten's equation. (15)

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