# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – APRIL 2017

## MT 1819- PROBABILITY THEORY & STOCHASTIC PROCESSES

Date: 05-05-2017 Dept. No. Max. : 100 Marks
09:00-12:00

**Section – A**
**Answer all the questions 10 x 2 = 20 marks**

1. If two fair dice are rolled find the probability of getting the sum to be at least 4.
2. If 4 marbles are chosen without replacement from an urn containing 6 blue 5 green and 4 yellow marbles , find the probability of getting at least 2 green marbles.
3. If $P(A) = ½$ , $P(B) = 1/3$ and $P(A \cap B) = 1/5$ find $P(A^c \cap B^c)$.
4. Define probability density function of a random variable.
5. If X is Poisson such that $P(X = 2) = P(X = 4)$ find mean of X.
6. If $f(x, y) = 2$ , $0 < x < y < 1$ , zero elsewhere , find the marginal p.d.f. of X.
7. Define gamma distribution with two parameters.
8. Define convergence in distribution.
9. Write a note on maximum likelihood estimation.
10. Define recurrence and periodicity of states of a Markov chain.

**Section – B**
**Answer any five questions**
**5 x 8 = 40 marks**

11. State and prove Boole's inequality.
12. If $f(x) = (x + 2)/18$ , $-2 < x < 4$ , zero elsewhere , find (i) $P(|X| < 2)$ (ii) $P(X^2 < 9)$.
13. Derive mean and variance of Poisson distribution.
14. Let $Y_1 < Y_2 < Y_3 \ldots < Y_n$ be the order statistics of a random sample of size n from a distribution with p.d.f. $f(x) = 1$ , $0 < x < 1$ , zero elsewhere. Show that the $k^{th}$ order statistic $Y_k$ has a beta p.d.f. with parameters $\alpha = k$ and $\beta = n - k + 1$.
15. If $X_1, X_2, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$ , find the maximum likelihood estimator of $\mu$.

16. Five measurements of the output of two units have given the following results(in kgs of material per one hour of operation):

Unit A : 14.1 10.1 14.7 13.7 14.0

Unit B : 14.014.5 13.7 12.7 14.1

Assuming that both samples have been obtained from normal populations, test at 1% level of significance if two populations have the same variance.

17. State and prove Chebyshev's inequality.

18. Show that the one dimensional random walk is recurrent if and only if p =q=1/2.

## Section – C

**Answer any two questions 2 x 20 = 40 marks**

19.(a) State and prove addition theorem on probability for n events. (8marks)

(b) State and prove Bayes' theorem. (6 marks)

(c) If f(x) = 2x , 0<x<1 , zero elsewhere , find (i) P(1/2 <X<3/4) (ii) P(-1/2 <X<1/2). Also find E(X). (6 marks)

20.(a) Derive the MGF of exponential distribution and hence find mean and variance. (10 marks)

(b) Let X be N($\mu$ , $\sigma^2$) so that P(X < 89) = 0.90 and P(X< 94) = 0.95 . Find $\mu$ and $\sigma^2$. (10 marks)

21.(a) Let $X_1$ and $X_2$ have the joint p.d.f. $f(x_1,x_2) = 2$ , $0 < x_1 < x_2 < 1$ , zero elsewhere. Find the conditional mean and variance of $X_1$ given $X_2 = x_2$ , $0<x_2<1$. (12 marks)

(b) Derive mean and variance of beta distribution of second kind. (8 marks)

22. (a) Fit a Poisson distribution to the following data test the goodness of fit at 5% level of significance.

| No. of errors/page: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| No. of pages : | 150 | 95 | 40 | 20 | 10 | 3 | 1 |

(10 marks)

(b) Derive Kolmogorov forward and backward differential equations for for birth and death process.

(10marks)

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