LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – APRIL 2017

MT 3501- ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date: 02-05-2017   Dept. No.   Max. : 100 Marks
09:00-12:00

PART – A

Answer all questions: **(10 X 2 = 20)**

1. Evaluate $\int\limits_{0}^{3}\int\limits_{1}^{2} xy(x+y)\,dy\,dx$.

2. Define Beta function..

3. Obtain a PDE by eliminating a and b from $z = ax + by + a$.

4. Solve $p = q^2$.

5. Prove that $\nabla.\vec{r} = 3$ where $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$.

6. Find the unit vector normal to the surface $x^2 + 2y^2 + z^2 = 7$ at $(1, -1, 2)$.

7. Find $L\left(t^2\right)$.

8. Find $L\left(\sin 3t\right)$.

9. Find the number of positive integers less than 720 and prime to it.

10. Define Euler's function.

PART – B

Answer any five questions: **(5 X 8 = 40)**

11. Evaluate $\int\int xy\,dx\,dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.

12. Prove that $\gamma(n + 1) = n!$ when n is a positive integer.

13. Solve $p^2 + q^2 = npq$.

14. Solve $p + q = x + y$.

15. Prove that $\nabla.r^n = nr^{n-2}\,\vec{r}$ where $\vec{r}$ is the position vector.

16. Find $L(te^{-t}\sin t)$.

1

17. Find $L^{-1}\left(\dfrac{1}{s(s+1)(s+2)}\right)$.

18. Find the number and sum of all the divisors of 360.

PART – C

Answer any two questions: **(2 X 20 = 40)**

19. (i) Prove that $\beta(m,n) = \dfrac{\Gamma m \; \Gamma(n)}{\Gamma(m+n)}$

    (ii) Prove that $\beta(m,n) = \beta(n,m)$.   **(15+5)**

20. (i) Find the general solution of $(y+z)p+(z+x)q = x+y$

    (ii) Obtain a complete integral of $xp^2 - ypq + y^3q - y^3z = 0$.   **(10+10)**

21. (i) Verify Gauss- Divergence theorem for the function $\vec{F} = 2xz\,\hat{i}+ yz\,\hat{j}+ z^2\,\hat{k}$ over the upper half

    of the sphere $x^2 + y^2 + z^2 = a^2$.

(ii) Find the highest power of 3 dividing 1000!   **(15+5)**

22. (i) Solve the equation $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 13y = 2e^{-x}$ given $y = 0, \dfrac{dy}{dx} = -1$ when $x = 0$.

    (ii) State and prove Wilson's theorem. **(10+10)**

****