LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRDSEMESTER – APRIL 2017

MT 3810- TOPOLOGY

Date: 06-05-2017 Dept. No. Max. : 100 Marks
01:00-04:00

Answer all the questions

I.a)1) Define the following: (i)topological space (ii) metrizable space and (iii) relative topology.
(3)

OR

a)2) Define the following in a topological space: (i) isolated point (ii) interior point and (iii) boundary point.
(3)

b)1) State and prove Cantor's intersection theorem.

b)2) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous $\Leftrightarrow f^{-1}(G)$ is open whenever G is open in Y.

(9+8)

OR

c)1) If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X then prove that there exists a point in X which is not in any of the A_n's.

c)2) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous at x_o if and only if $x_n \to x_o \Rightarrow f(x_n) \to f(x_o)$

(10+7)

II.a)1) Prove that any closed subspace of a compact space is compact.
(3)

OR

a)2) Define finite intersection property and state the equivalent form of the theorem: A topological space is compact iff every class of closed sets with empty intersection has a finite subclass with empty intersection.
(3)

b)1) State and prove Tychonoff's theorem.

b)2) State and prove the Generalized Heine-Borel theorem.

(9+8)

OR

c)1) State Lindelof's theorem.

c)2) Prove that a topological space is compact if every class of subbasic closed sets with the finite intersection property has non-empty intersection. (3+14)

III.a)1) Quoting the necessary results prove that every sequentially compact metric space is compact.
(3)

OR

a)2) Quoting the necessary results prove that a metric space is compact iff it is complete and totally bounded.(3)

b)1) Prove that a metric space is sequentially compact iff it has the Bolzano Weierstrass property.
b)2) State and prove Lebesgue's covering lemma.
(7+10)

OR

c) State and prove Ascoli's theorem.
(17)

IV.a)1) Prove that the range of a continuous real function defined on a connected space is an interval.
(3)

OR

a)2) Stating the results prove that R^n and C^n are connected.
(3)

b) State Urysohn imbedding theorem and prove Urysohn's lemma.
(17)

OR

c) State and prove Tietze Extension theorem.
(17)

V.a)1) Prove that X_∞ is compact.
(3)

OR

a)2) Explain the advantage of one point compactification to locally compact Hausdorff spaces.
(3)

b)State Complex Stone Weierstrass theorem. Prove the two lemmas that are required to prove its Real case.
(17)

OR

c) State and prove Weierstrass approximation theorem.
(17)

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