LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – APRIL 2017

MT 3812- CLASSICAL MECHANICS

Date: 28-04-2017   Dept. No.   Max. : 100 Marks
09:00-12:00

Answer ALL Questions.

1. (a) (i) Briefly explain D' Alembert's principle.

   OR

   (ii)   Discuss the motion of a simple pendulum.   (5)
   (b) (i) Derive the Lagrangian equation of motion.

   OR

   (ii) Discuss the equation of motion of a compound pendulum.   (15)

2. (a) (i) Derive the expression for Hamilton function H.

   OR

   (ii) Write down the Hamiltonian function and Hamiltonian's equations for a particle in a central force.   (5)

   (b) (i) State and prove the conservation theorem of energy.

   (ii) Find the Hamiltonian function for the motion of a projectile and hence deduce   the equation of motion.   (8+7)

   OR

   (iii) Derive the Hamilton's principle of least action.   (15)

3. (a) (i) Explain the Eulerian angles.

   OR

   (ii) Show that $Q = \tan^{-1}\left(\dfrac{q}{p}\right)$, $P=\dfrac{1}{2}\left(p^2 + q^2\right)$ represent a canonical transformation. (5)

   (b) (i) State and prove Invariant theorem of Poin Care.

   OR

   (ii) Discuss the motion of a top by Lagrange's method.   (15)

4. (a) (i) Show that $[u, vw] = v[u, w] + w[u, v]$.

   OR

   (ii) If u is a function of p, q, t, then prove that $\dfrac{du}{dt} = [u, H] + \dfrac{\partial u}{\partial t}$.(5)

   (b)(i) State and prove Conservation theorem of angular momentum.

   OR

   (ii) State and prove the Jacobi's identity.   (15)

5. (a) (i) Prove that $S = \int L dt + C$, where S if the function of motion and L the Lagrangian.

   OR

   (ii) Briefly explain Action-Angle variable.   (5)
   (b) (i) Discuss the Kepler's problem using Action angle variable.

   OR

   (ii) Derive Hamilton- Jacobi equation for the Hamilton's characteristic equation.   (15)

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