



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

FOURTH SEMESTER – APRIL 2017

MT 4200- ADVANCED MATHEMATICS FOR PHYSICS

(05 BATCH ONLY)

Date: 29-04-2017
09:00-12:00

Dept. No.

Max. : 100 Marks

SECTION A

Answer **ALL** the questions:

(10x2 = 20)

1. Evaluate $\int (x^4 + 3x) dx$.
2. Integrate $\int xe^x dx$ by using Bernoulli's formula.
3. Solve $\frac{dy}{dx} + \left(\frac{1-y^2}{1-x^2}\right)^{1/2} = 0$.
4. Define exact differential equation.
5. Evaluate $\int_0^2 \int_0^1 (x^2 y + y^2 x) dy dx$.
6. Prove that $\Gamma(n+1) = n\Gamma(n)$.
7. Find $\nabla\left(\frac{1}{r}\right)$, where $r = \sqrt{x^2 + y^2 + z^2}$.
8. State Stokes Theorem.
9. Define a Cyclic group and give an example.
10. Define contravariant and covariant vectors.

SECTION B

Answer any **FIVE** questions:

(5x8 = 40)

11. Evaluate $\int x^2 \tan^{-1} x dx$ using integration by parts method.
12. Prove that $\int_0^{\pi/4} \log[1 + \tan \theta] d\theta = \frac{\pi}{8} \log 2$.
13. Solve $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$ given that $y = 0$ when $x = 0$.
14. Solve $(D^2 + D + 1)y = x^2$.
15. Change the order of integration in the integral $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx dy$ and evaluate it.
16. If A_r^p and B_t^s are tensors, prove that $C_{rt}^{ps} = A_r^p B_t^s$ is also a tensor.
17. If $v = w \times r$, prove that $w = \frac{1}{2} \text{curl } v$, where w is a constant vector and r the position vector.
18. Show that the union of two subgroups of G is a subgroup if and only if one is contained in the other.

SECTION C

Answer any **TWO** questions:

(2x20 = 40)

19. (a) Evaluate $\int \frac{x}{\sqrt{x^2 + x + 1}} dx$.

(b) If

$$f(x) = \begin{cases} -x & \text{in } -\pi < x < 0 \\ x & \text{in } 0 \leq x < \pi \end{cases}, \text{ expand as Fourier series in the interval } -\pi \text{ to } \pi. \text{ Deduce}$$

$$\text{that } \frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$$

(8+12)

20. (a) Solve $(D^2 + 4D + 6)y = 5e^{-2x}$.

(b) Solve $(D^2 - 3D + 2)y = \sin 3x$.

(10+10)

21. (a) Find the Jacobian of x, y, z with respect to r, φ, θ where (r, φ, θ) are spherical coordinates.

(b) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(5+15)

22. (a) Show that if $F = x^3i + y^3j + z^3k$, $\iint_S F \cdot ndS = \frac{12}{5} \pi a^5$, where S is a sphere of radius a .

(b) Show that the set Z of all integers is a group with respect to the operation $*$ defined by $a * b = a + b + 1$ for all $a, b \in Z$.

(10+10)
