# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **PHYSICS**

FOURTH SEMESTER – APRIL 2017

**MT 4203- ADVANCED MATHEMATICS FOR PHYSICS**

**(07 BATCH ONWARDS)**

Date: 29-04-2017 Dept. No. Max. : 100 Marks
09:00-12:00

## PART A

**Answer ALL the questions.** **(10 × 2 = 20)**

1. Evaluate $\int \frac{dx}{4-9x^2}$.
2. Find the Fourier coefficient $a_0$ for $f(x) = x^2$ in the range $0 \le x \le 2\pi$.
3. State the necessary and sufficient condition for the ordinary differential equation to be exact.
4. Solve $(3D^2 + D - 14)y = 0$.
5. Integrate $\int_0^a \int_0^b (x^2 + y^2)dxdy$.
6. Evaluate $\int_0^{\frac{\pi}{2}} sin^7\theta cos^5\theta d\theta$.
7. Prove that the vector given by $\vec{F} = (x+3y)\vec{\imath} + (y-3z)\vec{\jmath} + (x-2z)\vec{k}$ is solenoidal.
8. For $\vec{F} = x^2y\vec{\imath} + xy^2z\vec{\jmath} - yz^2\vec{k}$, find $curl\vec{F}$.
9. Define a permutation group.
10. Define contravariant tensor of order 2.

## PART B

**Answer any FIVE questions.** **(5 × 8 = 40)**

11. Evaluate $\int \frac{x}{\sqrt{x^2+x+1}}dx$.
12. Prove that $\int_0^{\frac{\pi}{2}} \frac{(sinx)^{3/2}}{(sinx)^{3/2}+(cosx)^{3/2}}dx = \frac{\pi}{2}$.
13. Solve $xdy - ydx = \sqrt{x^2+y^2}$.
14. Evaluate $\iint r\sqrt{a^2-r^2}drd\theta$ taken over the upper half of the circle $r = acos\theta$.
15. Prove that $div\left(\frac{\vec{r}}{r^3}\right) = 0$.
16. If $\vec{F} = (2x+6y^2)\vec{\imath} - 10yz\vec{\jmath} + x^2z\vec{k}$, evaluate $\int_C \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t, y = t^2, z = t^3$.
17. Derive the relationship between Beta and Gamma functions.
18. Prove that every cyclic group is abelian.

## PART C

**Answer any TWO questions:** **(2 x 20 = 40)**

19. a) Evaluate $\int x^3 cos2x dx$

    b) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n\frac{cosnx}{n^2}$ in the interval $(-\pi \le x \le \pi)$. Also deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$. (5 +15)

20. a) Find the solution of the equation $\frac{dy}{dx} + ycosx = \frac{1}{2}sin2x$.

    b) Solve $(D^2 + 4D + 5)y = e^x + x^3 + cos2x$. (6 + 14)

21. a) By changing the order of integration evaluate $\int_0^a \int_{x^2/4a}^{2a-x} xydxdy$.

    b) Prove that the set of all residue classes of integers modulo 5 is an abelian group. (15 + 5)

22. Verify Gauss Divergence theorem, for $\vec{F} = 4xz\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ taken over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

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