LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTHSEMESTER – APRIL 2017

MT 4502- MODERN ALGEBRA

Date: 21-04-2017 Dept. No. Max. : 100 Marks
09:00-12:00

PART – A

ANSWER ALL QUESTIONS **(10 x 2 = 20)**

1. Define finite and infinite set.
2. Give an example of a finite group.
3. Define cyclic group and give an example.
4. How many generators are there for a cyclic group of order 9?
5. Define automorphism of a group with an example.
6. Define odd and even permutations.
7. Define an integral domain.
8. Define a field.
9. Define anEuclidena ring.
10. What is a Gaussian integer?

PART – B

ANSWER ANY FIVE QUESTIONS.

EACH QUESTION CARRIES EIGHT MARKS. **(5 x 8 = 40)**

11. Show that every group of order four is abelian.
12. If H and K are subgroups of G, then HK is a subgroup of G if and only if HK = KH.
13. Prove that every subgroup of a cyclic group is cyclic.
14. Let H be a subgroup of a group G. Then prove that any two left cosets of H in G are either identical or have no element in common.
15. If G is a group, then prove that A(G), the set of all automorphisms of G is a group.
16. Show that the intersection of two ideals of a ring R is an ideal of R.
17. Prove that every finite integral domain is a field.
18. Prove that the ring of integers is a PID.

PART – C

ANSWER ANY TWO QUESTIONS. **(2 x 20 = 40)**

19. a. If H and K are any two non – empty subsets of a group G, then prove that $(HK)^{-1} = K^{-1}H^{1}$.

    b. State and prove Lagrange's theorem.

20. a. Prove that any two infinite cyclic groups are isomorphic to each other.

    b. State and prove Cayley theorem.

21. a. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.

    b. If p is a prime then prove that Z_p is a field.

22. a. Prove that the characteristic of an integral domain D is either zero or a prime number.

    b. Prove that every Euclidean ring is a principal ideal domain.

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