# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **MATHEMATICS**

FIFTHSEMESTER – APRIL 2017

**MT 5405- FLUID DYNAMICS**

Date: 02-05-2017 Dept. No. Max. : 100 Marks
01:00-04:00

**Part A**

**Answer ALL Questions:** **(10 x 2 = 20)**

1. Define steady and unsteady flow.
2. Write down the boundary condition for the flow when it is moving.
3. Define path lines.
4. Prove that the fluid motion is possible if $\vec{q} = -Ay\vec{i} + Ax\vec{j}$ .
5. What is the complex potential of source with strength $m$ situated at the origin?
6. Find the stream function $\psi$, if $\varphi = A(x^2 - y^2)$ represents a possible fluid motion
7. Define the term complex potential.
8. Find the vorticity components of a fluid motion, if the velocity components are $u = c(x+y)$, $v = -c(x+y)$.
9. Define vortex filament.
10. What is Bernoulli's equation for steady ir-rotational flow?

**Part B**

**Answer ANY FIVE questions:** **($5\times 8 = 40$)**

11. Explain Material, Local and Convective derivative fluid motion.
12. The velocity $\vec{q}$ in a 3-dimensional flow field for an incompressible fluid is $\vec{q} = -3y^2\vec{i} - 6x\vec{j}$ .Determine the equation of streamlines passing through the point (1, 1, 1).
13. Derive the equation of continuity.
14. Let $\vec{q} = (Az - By)\vec{i} + (Bx - Cz)\vec{j} + (Cy - Ax)\vec{k}$, ($A$, $B$, $C$ are constants) be the velocity vector of a fluid motion. Find the equation of vortex lines.
15. Explain the construction of a Venturi tube.
16. Define path lines and determine the equation of path lines if $u = \frac{x}{1+t}, v = \frac{y}{1+t}, w = \frac{z}{1+t}$ .
17. Prove that for the complex potential $\tan^{-1} z$ the streamlines and equi - potentials are circles.
18. State and prove the theorem of Kutta-Joukowski.

**Part C**

**Answer ANY TWO questions:** **( $2\times 20 = 40$ )**

19. (a) For a two-dimensional flow the velocities at a point in a fluid may be expressed in the Eulerian coordinates by $u = x + y + 2t$ and $v = 2y + t$. Determine the Lagrange coordinates as functions of the initial positions $x_0$ , $y_0$ and the time $t$.

(b) Draw and explain the working of a Pitot tube. (12+8)

20. (a) What arrangement of sources and sinks will give rise to the function $w = \log(z - \frac{a^2}{z})$ ?

(b) If the velocity of an incompressible fluid at the point ($x$, $y$, $z$) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$ where $r^2 = x^2 + y^2 + z^2$. Prove that the fluid motion is possible and the velocity potential is $\frac{\cos\theta}{r^2}$. (8+12)

21. (a) Obtain the complex potential due to the image of a source with respect to a circle.

(b) Prove that the motion specified by $\vec{q} = \frac{k^2(x\vec{j} - y\vec{i})}{x^2 + y^2}$, ($k$ being a constant) is an irrotational flow. If so, find the velocity potential.

22. (a) Discuss the structure of an aerofoil.

(b) Derive Joukowski transformation. (8 +12)

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