LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – APRIL 2017

MT 5408- GRAPH THEORY

Date: 05-05-2017   Dept. No.   Max. : 100 Marks
09:00-12:00

PART – A

Answer ALL questions   **(10 x 2 = 20)**

1. Define a complete graph.
2. What is isolated vertex? Can you give an example?
3. Prove that $\delta \leq \frac{2q}{p} \leq \Delta$.
4. Define distance between any two vertices of a graph.
5. Define Hamiltonian graph with an example.
6. Define chromatic number.
7. What is eccentricity of a graph?
8. Define block with an example.
9. What is planar graph?
10. Show that $K_{3,3}$ is not planar.

PART – B

Answer any FIVE questions   **(5 x 8 = 40)**

11. a) Show that in any graph G the number of vertices of odd degree is even.

    b) Prove that $\Gamma(G) = \Gamma(\bar{G})$

12. Show that in a group of two or more people there are always two with exactly same number of friends inside the room

13. Define self complementary graph and prove that any self complementary graph has $4n$ or $4n+1$ vertices

14. If G is a graph in which the degree of every vertex is at least two then prove that G is contains a cycle.

15. If G is a graph with $p \geq 3$ vertices and $\delta \geq \frac{p}{2}$, then prove that G is Hamiltonian.

16. If G is connected planar graph having V, E and F as the set of vertices, edges and faces respectively then, prove that $|v| - |E| + |F| = 2$.

17. Prove that every non – trivial tree has atleast two vertices of degree one.

18.a) Prove that k_5 is not planar graph

b) If any connected (p, q) planar graph and $p \geq 3$ with r faces, prove that $q \geq \frac{3r}{2}$ and $q \leq 3p - 6$.

PART – C

19.(a) If Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then prove that
    (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph

    (ii) $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph

(b)The maximum number of edges among all p vertex graphs with no triangles is $\left[\frac{p^2}{4}\right]$.
 (6+ 14)

20.(a) Prove that a graph G with P vertices and $\delta \geq \frac{p-1}{2}$ is connected.

(b) Prove that a graph G with atleast two vertices is bipartite iff all its cycles are of even length.
                                        (5 +15)

21.(a) Let v be the vertex of a graph G,then prove that the following statement are equivalent.
         (i) v is a cut vertex of G.
         (ii) There exist partition of $G - \{v\}$ into v and w such that for each $u \in U$ and $w \in W$.
         (iii) There exist two vertices u, w distinct from v such that v is on every $u - w$ path

(b) Prove that every planar graph is five colourable.   (6 + 14)

22.(a) Let G be a (p, q) graph then prove that the following statements are equivalent
         (i) G is a tree.
         (ii) Every two points of G are joined by a unique path.
         (iii)G is connected and $p = q + 1$.
         (iv)G is acyclic and $p = q + 1$.

(b) If G is a connected (p, q) planar graph with no triangle and $p \geq 3$ then prove that
$q \leq 2p - 4$ (14 + 6)

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