LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – APRIL 2017

MT 5506- MECHANICS - I

Date: 24-04-2017 Dept. No. Max. : 100 Marks
01:00-04:00

PART – A

ANSWER **ALL** THE QUESTIONS: **($10 \times 2 = 20$ marks)**

1. State parallelogram of forces.
2. State perpendicular triangle of forces.
3. Define (a) Moment of a force and (b) Couple
4. Define (a) Limiting friction and (b) Dynamical friction.
5. Define angular velocity and angular acceleration.
6. A stone is dropped into a well and it reaches the bottom with a velocity of 96 per second and sound of splash of water reaches the top of the well in $3\frac{9}{70}$ seconds from the time the stone is dropped. Find the velocity of sound.
7. State Newton's laws of motion.
8. State the principle of conservation of linear momentum.
9. What is the time of flight of a projectile?
10. State Newton's experimental laws.

PART - B

ANSWER ANY **FIVE** QUESTIONS**:** **($5 \times 8 = 40$ marks)**

11. State and prove Lami's theorem.
12. The angle between two forces of magnitudes $P+Q$ and $P-Q$ is 2α and the resultant of forces makes an angle θ with the bisector of the angle between the forces. Show that $P\tan\theta = Q\tan\alpha$.
13. A uniform rod of length AB of length $2a$ and weight W is resting on two pegs C and D in the same level at a distance d are apart. The greatest weights that can be placed at A and B without tilting the rod are W_1 and W_2 respectively. Show that $\frac{W_1}{W+W_1} + \frac{W_2}{W+W_2} = \frac{d}{a}$.
14. A and B describe concentric circles of radii a and b with speeds u and v, the motion being the same way round. If the angular velocity of either with respect to the other is zero, prove that the line joining them subtends at the centre an angle whose cosine is $\frac{au+bv}{av+bu}$.
15. Discuss the motion of a particle moving along a straight line with uniform acceleration f.

16. Show that the velocity with which a particle must be projected down a smooth inclined plane of length l and height h so that the time of descent shall be the same as taken by another particle in falling freely through a distance equal to the height of the plane is $\frac{l^2-h^2}{l}\sqrt{\frac{g}{2h}}$

17. Show that the least velocity v required to hit a target at a height h and a horizontal distance a is given by the relation $v^2 = g\left[h+\sqrt{a^2+h^2}\right]$, where g is the acceleration due to gravity.

18. If A, B and C are 3 small smooth spheres of masses m, $2m$ and m respectively, lying in a straight line on a smooth horizontal table. A is projected along the line ABC with velocity u. If the coefficient of restitution be 0.5 in each case, show that after B strikes C, the velocities of A, B, C are in the ratio 0: 1: 2 and that there are no further impacts.

PART - C

ANSWER ANY **TWO** QUESTIONS: **($2 \times 20 = 40$ marks)**

19. (a) Two weights P and Q are suspended from a fixed point O by the strings OA and OB and are kept apart by a light rod AB. If the strings OA and OB make angles α and β with the rod, show that the angle θ which the rod makes with the vertical is given by $\tan\theta = \frac{P+Q}{Q\cot\beta - P\cot\alpha}$. **(10)**

    (b) A weight is supported on a smooth plane inclined at the angle α with the horizon, by a string inclined to the vertical at the angle β. If the inclination of the plane is increased to γ and the inclination of the string with the vertical is unaltered, the tension in the string is doubled in supporting the weight. Prove that $\cot\alpha - 2\cot\gamma = \cot\beta$. **(10)**

20. (a) Two like parallel forces P and $Q(P > 0)$ act at A and B respectively. If the magnitudes of the forces are interchanged. Show that the point of application of the resultant on AB will be displaced through the distance $\frac{P+Q}{P-Q}$ AB. **(08)**

    (b) State and prove Varignon's theorem on moments. **(12)**

21. (a) A car can acquire in one minute by uniform acceleration a speed of 30 mph. When it is halting at place in a straight narrow road, it sees another car approaching from behind with a uniform speed of 20 mph. Show that it will just be possible to avoid collision if the first car starts in full force before the second car approached it within $\frac{1}{9}$ of mile. **(08)**

    (b) Prove that the path of the projectile is a parabola. **(12)**

22. (a) Find the resultant of two like and unlike parallel forces. **(10)**

    (b) A particle projected upwards under the action of gravity in a resisting medium where the resistance varies as the square of the velocity. Discuss the motion. **(10)**

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