LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTHSEMESTER – APRIL 2017

MT 6606- COMPLEX ANALYSIS

Date: 20-04-2017 Dept. No. Max. : 100 Marks
09:00-12:00

PART - A

Answer **ALL** questions: **(10 X 2 = 20 marks)**

1. Verify Cauchy-Riemann equation for the function $f(z) = |z|^2$ at $z = 0$
2. Show that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic.
3. Define bilinear transformation.
4. Find the bilinear transformation which maps the points $z = 0, -i, -1$ into the points $w = i, 1, 0$ respectively.
5. Evaluate $\int_C \frac{1}{z}\, dz$ where C is the circle $|z| = r$.
6. Evaluate $\int_C \frac{\sin z}{\left(z - \frac{\pi}{2}\right)^2}\, dz$ where C is the circle $|z| = 2$.
7. Find the poles of the function $f(z) = \frac{1}{z(z-1)^2}$.
8. Define essential singularity.
9. Calculate the residue of $\frac{z+1}{z^2 - 2z}$ at its poles.
10. State Cauchy's residue theorem.

PART – B

Answer any **FIVE** questions: **(5 X 8 = 40 marks)**

11. If $\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$, prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$
12. If $f(z) = u(x, y) + iv(x, y)$ is an analytic function and $u(x, y) = \frac{\sin 2x}{\cosh 2y + \cos 2x}$ find $f(z)$.
13. Prove that any bilinear transformation preserves cross ratio.
14. State and prove maximum modulus theorem.
15. (a). State and prove Liouville's theorem. (4 marks)

    (b). Evaluate $\int_c \frac{z dz}{z^2 - 1}$ where C is the positively oriented circle $|z| = 2$. (4 marks)
16. Find the Taylor's series to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| < 2$.
17. State and prove Riemann's theorem.
18. State and prove Argument theorem.

PART – C

Answer any **TWO** questions: **(2 X 20 = 40 marks)**

19. (a). Derive C.R. Equations in Polar Co-ordinates. **(10 marks)**

    (b). State and prove Cauchy's Integral formula. **(10 marks)**

20. (a). Prove that a bilinear transformation w = $\frac{az+b}{cz+d}$ where ad − bc ≠ 0 maps the realaxis into itself if and only if a, b, c, d are real. **(10 marks)**

    (b).Show that the transformation $w = \frac{5-4z}{4z-2}$ maps the unit circle $|Z| = 1$ into a circle of radius unity and centre $\frac{-1}{2}$ **(10 marks)**

21. State and prove Laurent's theorem. **(20 marks)**

22. (a). State and prove Rouche's theorem. **(10 marks)**

    (b). Evaluate $\int_C \tan z \, dz$ where C is $|z| = 2$ using residue theorem. **(10 marks)**

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