LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **APRIL 2018**

16UMT3MC02– VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 07-05-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL Questions: $\mathbf{10 \times 2 = 20}$

1. If $\emptyset(x,y,z) = x^2y + y^2x + z^2$ find $\nabla\emptyset$ at the point $(1,1,1)$.
2. Prove that $div\,\vec{r} = 3$ and $curl\vec{r} = 0$ where $\vec{r}$ is the position vector of the point (x,y,z).
3. Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative vector field.
4. If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, evaluate $\int_C \vec{F}.\,d\vec{r}$ where C is the curve on the xy plane $y = 2x^2$ from (0,0) to (1,2).
5. State Green's theorem in plane.
6. State Stoke's theorem.
7. Solve $\frac{dy}{dx} + \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} = 0$.
8. Find the general solution of $y = xp + \frac{a}{p}$.
9. Solve $(D^2 - 4D + 3)y = 0$.
10. Find the particular integral of $(D^2 - 6D + 9)y = e^{3x}$.

PART – B

Answer ANY FIVE Questions: $\mathbf{5 \times 8 = 40}$

11. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}| = r$, prove that (i) $\nabla r = \frac{\vec{r}}{r}$ (ii) $\nabla(\log r) = \frac{\vec{r}}{r^2}$.
12. Find the maximum value of the directional derivative of the function $\emptyset = 2x^2 + 3y^2 + 5z^2$ at the point $(1,1,-4)$.
13. Evaluate $\iint_S \vec{F}.\vec{n}ds$ where $\vec{F} = z\vec{i} + x\vec{j} - y^2z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$ included in the first octant between $z = 0$ and $z = 2$.

14. Evaluate $\int_C (x^2+y^2+z^2)ds$, where C is the arc of the circular helix

$x = 3cost, y = 3sint, z = 4t$ from $A(3,0,0)$ to $B(3,0,8\pi)$.

15. By Green's theorem, find the value of $\int_C (x^2ydx + ydy)$ along the closed curve C formed by

$y^2 = x$ and $y = x$ between (0,0) and (1,1).

16. Solve $xdy - ydx = \sqrt{x^2+y^2}dx$.

17. Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$.

18. Solve $(D^2 - 8D + 9)y = 8sin5x$.

PART – C

Answer ANY TWO Questions: $\mathbf{2 \times 20 = 40}$

19. a) Prove that for any vector $\vec{A}$, $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2.\vec{A}$

b) Evaluate $\iiint_V \vec{F}dV$ if $\vec{F} = 2xz\vec{\imath} - x\vec{\jmath} + y^2\vec{k}$ and V is the volume enclosed by the cylinder

$x^2 + y^2 = a^2$ between the planes $z = 0$ and $z = c$. **(10+10)**

20. Verify the Gauss Divergence theorem for the function $\vec{F} = 2xz\vec{\imath} + yz\vec{\jmath} + z^2\vec{k}$ taken over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$

21. a) Solve $\frac{dy}{dx} + ycosx = \frac{1}{2}sin2x$.

b) Solve $x^2p^2 + 3xyp + 2y^2 = 0$. **(8+12)**

22. a) Solve $(D^2 + 4)y = xsinx$.

b) Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + logx$. **(8+12)**

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