LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – APRIL 2018

17PMT1MC02- REAL ANALYSIS

Date: 24-04-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answerallquestions.All questions carry equal marks

1. (a) (i) State and prove mean value theorem.

OR

(ii) If f is a real valued function defined on $[a, b]$, f has local maximum at a point $x \in [a, b]$ and $f'(x)$ exists, then prove that $f'(x) = 0$. (5 marks)

(b) (i) Suppose f is continuous on $[a, b]$, $f'(x)$ exists at some point $.x \in [a, b]$, g is defined on an interval I which contains the range of f and g is differentiable at the point f(x). If $h(t) = g\big(f(t)\big), a \leq t \leq b$, then prove that *h* is differentiable at *x* and $h'(x) = g'\big(f(x)\big)f'(x)$. (9 marks)

(ii) If f is a continuous mapping of a metric space X into a metric space Y and E is a connected subset of X, then prove that f (E) is connected. (6 marks)

OR

(c) (i) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y. (7 marks)

(ii) Suppose f is a real differentiable function on [a,b] and suppose $f'(a) < \lambda < f'(b)$. Prove that there is a point x$\in (a, b)$ such that $f'(x) = \lambda$. (8 marks)

2. (a) (i) Define a refinement of a partition P. If P^* is a refinement of P then prove that $L(P, f, \propto) \leq L(P^*, f, \propto)$ and $U(P^*, f, \propto) \leq U(P, f, \propto)$.

OR

(ii) If $f \in \Re(\alpha)$ and $g \in \Re(\alpha)$ on [a, b], then prove that $|f| \in \Re(\alpha)$ and $\left|\int_a^b f d\alpha\right| \leq \int_a^b |f| d\alpha$. (5 marks)

(b) State and prove a necessary and sufficient conditions for a bounded real valued function to be a Riemann-Steiltjesintegrable. (15 marks)

OR

(c) (i) State and prove the theorem on Integration by parts.

(ii) If f is a real continuously differentiable function on [a, b] with f (a) = f (b) = 0 and $\int_a^b f^2(x)dx = 1$, then prove that $\int_a^b xf(x)f'(x)dx = -\frac{1}{2}$ (5+10 marks)

3. (a) (i) Prove that for $f_n(x) = n^2x(1-x^2)^n, 0 \leq x \leq 1, n = 1,2 \ldots,$ $\int_0^1 \left(\lim_{n\to\infty} f_n(x)\right) dx \neq \lim_{n\to\infty} \int_0^1 f_n(x)dx$.

OR

(ii) Suppose $\{f_n\}$ is a sequence of functions on a set E and $|f_n(x)| \leq M_n, x \in E, n = 1,2 \ldots$ then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ Converges. (5 marks)

(b) If $\{f_n\}$ is a sequence of continuous functions on a set E and if $f_n \to f$ uniformly on E, then prove that f is continuous on E.

OR

(c) State and prove the Stone-Weierstrass theorem. (15 marks)

4. (a) (i) State and prove the Bessel's Inequality and hence derive the Parseval's formula.

OR

(ii) State and prove the Riesz-Fischer theorem. (5 marks)

(b) State and prove the Riemann-Lebesgue lemma and use the lemma to prove the following:

For $f \in L(-\infty, +\infty)$, $\lim_{\propto \to \infty} \int_{-\infty}^{\infty} f(t) \frac{1-\cos\propto t}{t} dt = \int_{0}^{\infty} \frac{f(t)-f(-t)}{t} dt.$ (15 marks)

OR

(c) (i) If g is of bounded variation on $[0, \delta]$, then prove that

$$\lim_{\alpha \to \infty} \frac{2}{\pi} \int_0^{\delta} g(t) \frac{\sin\alpha t}{t} dt = g(0+).$$

(ii) If $f \in L[0,2\pi]$, f is periodic with period 2π and $\{s_n\}$ is a sequence of partial sums of Fourier series generated by f, $s_n = \frac{a_0}{2} + \sum_{k=1}^{n} (a_k \cos kx + b_k \sin kx)$,

$n = 1,2$ ...then prove that $s_n(x) = \frac{2}{\pi} \int_0^{\pi} \frac{f(x+t)+f(x-t)}{2} D_n(t) dt$ (8+7 marks)

5. (a) (i) State and prove the fixed point theorem.

OR

(ii) If Ω is the set of all invertible linear operators on R^n and for $A \in \Omega, B \in L(R^n)$, if $||B - A|| \, ||A^{-1}|| < 1$, then prove that $B \in \Omega$. (5 marks)

(b) State and prove the inverse function theorem.

OR

(c) State and prove the implicit function theorem. (15 marks)

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