LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

SECONDSEMESTER – APRIL 2018

17/16PMT2MC04- COMPLEX ANALYSIS

Date: 23-04-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions.

1. a. Evaluate $\int_\gamma \frac{e^{iz}}{z^2} dz, \gamma(t) = e^{it}, 0 \le t \le 2\pi$.

OR

b. State and prove Liouville's theorem. (5 marks)

c. Prove that any complex valued differentiable function defined on an open set G is analytic on G.

OR

d. State and prove homotopic version of Cauchy's theorem. (15 marks)

2. a. If $\gamma: [0,1] \to C$ is a closed curve and $a \notin \{\gamma\}$, then prove that $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is an integer.

OR

b. State and prove Morera's theorem. (5 marks)

c. State and prove Schwarz lemma. Also prove that if $f: D \to D$is a one-one analytic map of D onto itself and f (a) =0, then prove that there is a complex number c with $|c| = 1$ such that $f = c\varphi_a$.

OR

d. State and prove the Riemann mapping theorem. (15 marks)

3. a. For $Rez_n > 1$, prove that the series $\sum_{n=1}^{\infty} \log(1 + z_n)$ converges absolutely if and only if the series $\sum_{n=1}^{\infty} z_n$ converges absolutely.

OR

b. State and prove Functional equation. (5 marks)

c. State and prove the Weierstass factorization theorem.

OR

d. State and prove Bohr-Mollerup theorem. (15 marks)

4. a. Prove that $sin\pi z = \pi z \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right)$.

OR

b. State and prove MittagLeffler's theorem.

c. State and prove Hadamard's factorization theorem.

OR

d. If f is an entire function of finite genus μ, then prove that f is of finite order λ and $\lambda \leq \mu + 1$.

(15 marks)

5. a. Prove that a non – constant elliptic function has equally many poles as it has many zeros is a period parallelogram.

OR

b. Derive Legendre relation. (5 marks)

c. (i) Derive the relation $\mathcal{Z}(z) = \frac{1}{z} + \sum_{\omega \neq 0} \frac{1}{z-\omega} + \frac{1}{\omega} + \frac{z}{\omega^2}$.

(ii) Prove that any two bases of the same module are connected by a unimodular transformation. (10+5)

OR

d) (i) Derive the first order differential equation satisfied by $\mathcal{P}(z)$.

ii) Prove the following: $\sigma(z+\omega_1) = -\sigma(z)e^{n_1\left(z+\frac{\omega_1}{2}\right)}$and $\sigma(z+\omega_2) - \sigma(z)e^{n_2\left(z+\frac{\omega_2}{2}\right)}$.

(10+5)