LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

SECONDSEMESTER – APRIL 2018

17/16UMT2AL01- MATHEMATICS FOR PHYSICS - II

Date: 28-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Part A (Answer ALL questions) **(10X2 =20)**

1. Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$.
2. Find the value of $\int_0^{\pi/2} \cos^6 x\, dx$
3. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
4. Write any two properties of beta function.
5. Solve $x\sqrt{1+y^2} + y\sqrt{1+x^2}\frac{dy}{dx} = 0$.
6. Show that the following differential equation is exact.
   $(x^2 - x + y^2)dx - (ye^y - 2xy)dy = 0$
7. Find $\frac{\partial(u,v)}{\partial(x,y)}$ when $u = x + y$ and $v = x - y$.
8. Evaluate $\int_0^1\int_0^2\int_0^3 xyz\, dxdydz$.
9. Prove that $\nabla \cdot r = 3$ and $\nabla \times r = 0$ where r is the position vector of the point P(x,y,z).
10. State Stoke's theorem.

Part B (Answer any FIVE questions) **(5 x 8 = 40)**

11. Evaluate $\int \frac{(3x+1)dx}{(x-1)^2(x+3)}$.
12. Establish the reduction formula for $I_n = \int \sin^n x dx$ (n being a positive integer)and hence find the value of $\int_0^{\pi/2} \sin^5 x\, dx$.
13. Evaluate $I = \int_0^{\pi/2} \log\sin x\, dx$

14. Prove the following.

i. $\Gamma(n+1) = n!$, where n is a positive integer.

ii. $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

15. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\cos 2x$

16. Solve $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

17. By transforming into polar co-ordinates evaluate $\iint \frac{x^2y^2}{x^2+y^2}dxdy$ over the annular region between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ $(b>a)$

18. Evaluate $\iint_S F\cdot\hat{n}\, dS$, where F = zi +xj –y^2 z k, and S is the surface of the cylinder x^2 +y^2 =1 included in the first octant between the planes z=0 and z=2.

Part C (Answer any TWO questions) **(2 x 20 = 40)**

19. a) Prove that $\int_0^{\pi/4} \log(1+\tan\theta)d\theta = \frac{\pi}{8}\log 2$

b) Solve $(D^2+16)y = 2e^{-3x} + \cos 4x$

(10 + 10)

20. a) Express $I = \int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma function and evaluate the integral

$$I = \int_0^1 x^5(1-x^3)^{10} dx$$

b). Evaluate $\int_0^1 x^m(\log\frac{1}{x})^n dx$ (10 + 10)

21.a) By changing the order of integration , evaluate $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y} dxdy$

b) By changing into polar co-ordinates evaluate the integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} (x^2+y^2)dxdy$ (8 +12)

22. a) Evaluate $\int \frac{6x+5}{\sqrt{6+x-2x^2}}dx$

b) Find by Green's theorem the value of $\int_C (x^2ydx + ydy)$ along the closed curve C formed by the curves y^2 =x and y=x between (0, 0) and (1, 1). (10+10)