LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION –MATHEMATICS

FIRST SEMESTER – APRIL 2018

17/16PMT1MC03– ORDINARY DIFFERENTIAL EQUATIONS

Date: 28-04-2018   Dept. No.   Max. : 100 Marks
Time: 09:00-12:00

Answer all questions. Each question carries 20 marks.

1. (a) Determine whether the given sets of functions are linearly dependent or independent.
     (i) $\{e^x, e^{-x}\}$,   (ii) $\{1+x,\ 1-x, 1-3x\}$.   (5)

     (OR)

   (b) State and prove Abel's formula.   **(5)**
   (c) Using the method of variation of parameters, solve $x'''(t) - x'(t) = e^t$.   **(15)**

     (OR)

   (d) Derive the various solutions of the second order linear homogenous differential equation with constant coefficients.   **(15)**

2. (a) State and prove Rodrigue's formula.   **(5)**

     (OR)

   (b) With usual notations, prove that (i) $P_l(1) = 1$, and (ii) $P_l'(1) = l(l+1)/2$.   **(5)**
   (c) Solve $2x\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ by Frobenius method.   **(15)**

     (OR)

   (d) Derive the orthogonality properties of the Legendre's polynomial.   **(15)**

3. (a) Obtain the generating function of Bessel's function.   **(5)**

     (OR)

   (b) When n is a non-zero integer, prove that $J_{-n}(x) = (-1)^n J_n(x)$.   **(5)**

   (c) Let $J_n(x)$ be the Bessel's function of first kind of order n and $Y_n(x) = \frac{\cos n\pi\, J_n(x) - J_{-n}(x)}{\sin n\pi}$. Prove that the two independent solutions of the Bessel's equation may be taken to be $J_n(x)$ and $Y_n(x)$ for all values of n.   **(15)**

     (OR)

   (d) Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0, n \geq 0$.   **(15)**

4. (a) Prove that all the eigenvalues of Strum-Liouville problem are real.   **(5)**

     (OR)

   (b) State and prove Grownwall inequality.   **(5)**
   (c) State and prove Picard's theorem for initial value problem.   **(15)**

     (OR)

   (d) Prove that $x(t)$ is a solution of the equation $L[x(t)] + f(t) = 0, a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$ where $G(t,s)$ is the Green's function.   **(15)**

5. (a) Explain asymptotically stable solution by an example.   **(5)**

     (OR)

   (b) Define an autonomous system and state the stability behaviours of the system.   **(5)**
   (c) Discuss the stability of linear system $x' = Ax$ using Lyapunov's function.   **(15)**

     (OR)

   (d) State and prove the two fundamental theorems on the stability of non-autonomous systems.
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