LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**MATHEMATICS** **[*******]**

FIRST SEMESTER – **APRIL 2018**

17/16UMT1MC02– ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 26-04-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer ALL questions: **(10 X 2 = 20)**

1. Find $\lim_{x\to 0}\frac{\sin x}{x}$.

2. Write the binomial expansion of $(\cos\theta + i\sin\theta)^5$.

3. Prove that $\sin(ix) = i\sinh x$.

4. Show that $\cosh^2 x - \sinh^2 x = 1$

5. Find the eigen values of the matrix $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$.

6. State Cayley- Hamilton theorem.

7. Write the polar of any point (x_1, y_1) and the pole of the line $Ax + By + C = 0$ with respect to the parabola $y^2 = 4ax$

8.Define conjugate diameters of the ellipse.

9. Show that the sum of the squares of two conjugate semi diameters of an ellipse is constant.

10. Write the polar equation of a straight line.

PART – B

Answer any FIVE questions: **(5 X8 = 40)**

11. Expand $\sin^3\theta\cos^5\theta$ in terms of multiples of θ.

12. If $\frac{\tan\theta}{\theta} = \frac{2524}{2523}$ prove that $\theta = 1^{o}58'$ approximately.

13. Prove that $\sinh^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$

14. Separate into real and imaginary parts $\tan^{-1}(x+iy)$

15. Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$.

16. Find the locus of the poles all tangents to the parabola $y^2 = 4ax$ with respect to the parabola $y^2 = 4bx$.

17. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

18. Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a\cos\theta$.

PART– C

Answer Any TWO Questions: **(2 X 20 = 40)**

19. a. Expand $\cos 8\theta$ in terms of sines of multiples of θ

b. Evaluate $\lim_{x\to 0}\frac{\tan 2x - 2\tan x}{x^3}$. **(10 + 10)**

20. a.Ifcos (x + iy) = cosα + isinα , prove that cos 2x + cosh 2y = 2

b. Reduce $(x+i\beta)^{x+iy}$ $to\, the\ form\, A+iB$. **(10 + 10)**

21. Diagonalize the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

22. a. If e, e_1 are the eccentricities of a hyperbola and its conjugate show that $\frac{1}{e^2}+\frac{1}{e_1^{\,2}}=1$

b. Trace the curve $\frac{12}{r}=4+\sqrt{3}\cos\theta+\sin\theta$ **(10 + 10)**

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