# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc.** DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2018**

**17/16UMT2AL02– MATHEMATICS FOR STATISTICS- II**

Date: 28-04-2018 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

## SECTION A

**ANSWER ALL QUESTIONS.** **(10 × 2 = 20)**

1. Write the formula for $S_n$ for the following sequences.
   (i) $1, -4, 9, -16, \ldots$
   (ii) $1, 3, 6, 10, \ldots$
2. Define least upper bound and greatest lower bound.
3. Define conditional convergence of a series.
4. Give examples of convergent and divergent series.
5. Define limit of a function on the real line.
6. When you say the subset $D$ of $R$ is of first category?
7. Verify the function $f(x) = sinx\ (0 \leq x \leq \pi)$ obey the hypothesis of Rolle's theorem.
8. Prove that the derivative of a constant function on $[a, b]$ is the identically zero function on $[a, b]$.
9. Define Riemann integral.
10. What is meant by a subdivision of the closed bounded interval $[a, b]$?

## SECTION B

**ANSWER ANY FIVE QUESTIONS.** **(5 × 8 = 40)**

11. If $A, B$ are subsets of $S$, then prove that (i) $(A \cup B)' = A' \cap B'$ and (ii) $(A \cap B)' = A' \cup B'$.
12. If $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} s_n = M$, then prove that $L = M$.
13. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then prove the following:
$(i)\ \lim_{x\to a}[f(x) + g(x)] = L + M, (ii)\ \lim_{x\to a}[f(x). g(x)] = L . M$ .
14. Let $f: R \to R$ and let $D$ be the set of points at which $f$ is not continuous, then prove that $D$ is of type $F_\sigma$.
15. Define Continuous and derivative of a real valued function $f$ at $c$ and hence prove that if $f$ has a derivative at the point $c \in R$, then $f$ is continuous at $c$.
16. Define sets of measure zero and hence prove that if each of the subsets $E_1, E_2, \ldots$ of R is of measure zero, then $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.
17. Let $f$ be a bounded function on $[a, b]$. If $\sigma$ and $\tau$ are any two subdivisions of $[a, b]$, then prove that $U[f: \sigma] \geq L[f: \tau]$.
18. State and prove Binomial theorem.

## SECTION C

**ANSWER ANY TWO QUESTIONS.** **(2 x 20 = 40)**

19. (a) Prove that $\lim_{n\to\infty} \frac{3n^2-6n}{5n^2+4} = \frac{3}{5}$.

(b) Define bounded sequence and hence prove that if the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$is convergent then $\{s_n\}_{n=1}^{\infty}$ is bounded.

(c) If $R_{n+1}(x) = \frac{1}{n!}\int_a^x f^{(n+1)}(t)\,(x-t)^n dt$, then prove that

$R_n(x) - R_{n+1}(x) = \frac{f^{(n)}(a)(x-a)^n}{n!}$. **(5+7+8)**

20. (a) State and prove Comparison test for series.

(b) If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then prove tha$\sum_{n=1}^{\infty}(a_n + b_n)$ converges to $A$+$B$. **(12+8)**

21. (a) State and prove Rolle 's Theorem.

(b)Let $f$ and $g$ be continuous functions on the closed bounded interval $[a,b]$ with$g(a) \neq g(b)$. If both $f$ and$g$ have a derivative at each point of $(a,b)$ and $f'(t)$ and $g'(t)$ are not both equal to zero for any $t \in (a,b)$, then prove that there exists a point $c \in (a,b)$ such that $\frac{f'(t)}{g'(t)} = \frac{f(b)-f(a)}{g(b)-g(a)}$.

**(12+8)**

22. (a) If $f, g \in \mathfrak{R}[a,b]$, then prove that $f + g \in \mathfrak{R}[a,b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

(b) Let $f$ be a bounded function on the closed bounded interval $[a,b]$, then prove that $f \in \mathfrak{R}[a,b]$if and only if for each $\varepsilon > 0$, there exists a subdivision $\sigma$ of $[a,b]$ such that $U[f:\sigma] < L[f:\sigma] + \varepsilon$. **(12+8)**

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