LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**MATHEMATICS**

SECOND SEMESTER – **APRIL 2018**

17/16UMT2MC02 – ANA. GEO. OF 3D, FOURIER SERIES AND NUMBER THEORY

Date: 26-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART-A

Answer ALL questions: **(10 x 2=20)**

1. Find the equation to the plane through (3, 4, 5) parallel to the plane 2x+3y-z+2 = 0.
2. State the equation of line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2).
3. Find the equation to the sphere whose centre is (2, -3, 4) and radius is 5 units.
4. Write the equation of the tangent plane to the sphere.
5. Define Fourier series.
6. Is the function $f(x) = \sin 2x$ odd? Justify your answer.
7. Find the number of divisors of 840.
8. Find the number of integers less than 729 and prime to it.
9. If x, y, z be real and not all equal, show that $(x + y + z)(yz + zx + xy) > 9xyz$.
10. State Cauchy's inequality.

PART - B

Answer any FIVE questions: **(5 X 8 = 40)**

11. Show that, if a plane has intercepts *a*, *b*, *c* on the coordinate axes and is at a distance *p* from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.
12. Find the shortest distance between the line $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$.
13. Find the condition that the plane lx + my + nz = p may be a tangent plane to the sphere $x^2 + +y^2 + z^2$ + 2ux + 2vy + 2wz + d = 0.
14. Find the equation of the sphere through the points (2,3,1), (5, -1, 2), (4, 3, -1) and (2, 5, 3).
15. Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series with period 2π, to be valid in the interval 0 to 2π.
16. Find a sine series for $f(x) = \begin{cases} x \quad when\ 0 < x < \frac{\pi}{2} \\ 0 \ \ when\ \frac{\pi}{2} < x < \pi \end{cases}$.
17. Find the remainder obtained in dividing 2^{46} by 47.
18. Show that the 8th power of any number is of the form 17m or 17m$\pm$1.

PART – C

Answer any TWO questions: **(2 x 20=40)**

19. (a) Find the equation of the plane through the line of intersection of the planes x + y + z = 1, 2x +3y + 4z – 7 = 0 and perpendicular to the plane x - 5y + 3z = 5.

(b) Find the equation of the image of the line $\frac{x-1}{2} = \frac{y+2}{-5} = \frac{z-3}{2}$ in the plane $2x - 3y + 2z + 3 = 0$.

(10+10)

20. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Find the equation of the circumcircle of the triangle and determine the coordinates of the centre and radius.

21. Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{cosnx}{n^2}$ in the interval $-\pi \leq x \leq \pi$. Deduce the sum of the series (i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$, (ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$.

22. (a) State and prove Wilson's Theorem.

(b) Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22. **(10+10)**

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