LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**MATHEMATICS**

FIFTH SEMESTER – **APRIL 2018**

MT 5505– REAL ANALYSIS

Date: 30-04-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

SECTION- A

Answer all the questions: 2x10=20

1. State the principle of mathematical induction.

2. State the triangle inequality.

3. Define a metric space.

4. Define a complete metricspace with an example.

5. State intermediate value theorem.

6.What is a homeomorphism?

7. Define a local maximum for a function at a point.

8. State Lagrange's mean value theorem.

9. Define a bounded variation of a function.

10. State the linearity properties of Riemann Steiltjes integral.

SECTION –B

Answer any FIVE questions: 5 X 8= 40

11. Prove that every subset of a countable set is countable.

12. State and prove Cauchy-Schwarz inequality.

13. Prove that every convergent sequence is Cauchy.

14. Let (X, d_1) and (Y, d_2) be metric spaces and $f : X \to Y$ be a continuous function on X. If X is compact, then prove that f(X) is a compact subset of Y.

15. If $f : X \to Y$ is continuous on X and if X is compact, then prove that f is Uniformlycontinuous.

16. State and prove the intermediate value theorem for derivatives.

17. If f is a monotonic function on [a, b], then prove that the set of discontinuities of f is countable.

18. State and prove the theorem on Integration by parts.

SECTION –C

Answer any TWO questions: **2 X 20= 40**

19.a) Prove that the set **R** is uncountable.

b) State and prove Minkowski's inequality.

20.a) Let Y be a subspace of a metric space (X, d). Then prove that a subset A of Y is open in Y

if and only if $A = Y \cap G$ for some set G open in X.

b) Prove that the Eulideanspace R^k is complete.

21. a) Let (X, d_1) and (Y, d_2) be metric spaces and $f : X \to Y.$ If x_0 belongs to X , then prove

that**f** is continuous at x_0 if and only if for every sequence $\{x_n\}$ in X that converges to x_0, the

sequence $\{f(x_n)\}$ converges to $f(x_0)$

b) If f and g are continuous at x_0 and k is a fixed real number, then prove that (i) kf , (ii) f + g

and (iii) fg are continuous at x_0 in X.

22. a) State and prove Rolle's theorem.

b) If $f \in R(\alpha)$ on [a, b] and $f \in R(\beta)$ on [a, b] then prove that for any pair of constants λ and

μ the following are true. (i) $f \in R(\lambda\alpha + \mu\beta)$ on [a, b] and

(ii) $\int_a^b f d(\lambda\alpha + \mu\beta) = \lambda \int_a^b f d\alpha + \mu \int_a^b f d\beta$

****