LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2018**

MT 1500– ALGEBRA, ANALY. GEO., CALCULUS & TRIGONOMETRY

Date: 25-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Section – A (10 x 2 = 20 Marks)

Answer all the questions :

1) Write down the n^{th} derivative of y = log (ax + b).
2) Show that in a parabola $y^2 = 4ax$ the subnormal is constant.
3) Define evolute of a curve.
4) Write the formula for radius of curvature when the curve is given in polar coordinates.
5) Define reciprocal equation.
6) Diminish roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$.
7) Show that $\cos h^2 x - \sin h^2 x = 1$
8) Write down the expansion for $\tan n\theta$.
9) If e_1 and e_2 are eccentricities of a hyperbola and its conjugate then prove that $e_1^2 + e_2^2 = 1$
10) Write the angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Section – B (5 x 8 = 40 Marks)

Answer any five questions :

11) Find the n^{th} derivative of $y = \frac{x+1}{(2x-1)(2x+3)}$.
12) Prove that the radius of a curvature at the point ($a\cos^3\theta$, $a\sin^3\theta$) on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $3a \sin\theta \cos\theta$.
13) Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$ given that $2 + i\sqrt{3}$ is a root.
14) If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$ find the equation whose roots are $\alpha^2, \beta^2, \gamma^2$
15) Prove that $\frac{Sin\ 7\theta}{Sin\ \theta} = 64\ Cos^6\theta - 80\ Cos^4\theta + 24\ Cos^2\theta - 1$
16) If $\tan(\propto + i\beta) = x + iy$, Prove that $x^2 + y^2 + 2x \cot 2\alpha + 1 = 0$.
17) Prove that eccentric angles of the ends of a pair of conjugate diameters differ by a right angle.
18) Derive the polar equation of a conic.

Section – C (2 x 20 = 40 Marks)

Answer any two questions :

19) a) If $y = \sin(m \sin^{-1} x)$,
prove that $(1-x^2)y_{n+2} - (2n+1)\, xy_{n+1} + (m^2 - n^2)y_n = 0$.

b) Find the angle of intersection of the curves $r = \frac{a}{1+Cos\theta}$ and $r = \frac{b}{1-Cos\theta}$.

20) a) Find the evolute of the parabola $y^2 = 4ax$.
b) Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

21) a) Show that if the roots of the equation $x^3 + px^2 + qx + r = 0$ are in the A.P then $2p^3 - 9pq + 27r = 0$.
b) Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to 2 decimals by Horner's method.

22) a) If $\sin(A+iB) = x + iy$, prove that

i) $\frac{x^2}{cosh^2 B} + \frac{y^2}{\sin h^2 B} = 1$

ii) $\frac{x^2}{sin^2 A} - \frac{y^2}{cos^2 A} = 1$

b) Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

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