LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2018**

MT 1502– ALGEBRA AND CALCULUS - I

Date: 25-04-2018   Dept. No.   Max. : 100 Marks
Time: 01:00-04:00

PART A

ANSWER ALL THE QUESTIONS   ($10 * 2 = 20 marks$)

1. Find the nth derivative of e^{ax}.

2. Show that in the curve $r = a\theta$ the polar subtangent varies as the square of the radius vector.

3. Write the conditions for the maxima and minima of functions of two variables.

4. Write down the use of Lagrange's multipliers' method.

5. Write the Cartesian formula for the radius of curvature.

6. Write down a pedal equation of a curve.

7. Form a rational cubic equation which shall have for roots $1, 3 - \sqrt{-2}$.

8. Define reciprocal equation.

9. Define Descartes' rule of signs for negative roots..

10. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ form the equation whose roots

    are $\alpha - \dfrac{1}{\beta\gamma}, \beta - \dfrac{1}{\gamma\alpha}, \gamma - \dfrac{1}{\alpha\beta}$

PART B

ANSWER ANY FIVE QUESTIONS   ($5 * 8 = 40 marks$)

11. Find the nth differential coefficient of $x^2 log x$.

12. Find the angle at which the radius vector cuts the curve $\dfrac{l}{r} = 1 + e\cos\theta$.

13. Find the maximum or minimum values of $2(x^2 - y^2) - x^4 + y^4$.

14. What is the radius of curvature of the curve $x^4 + y^4 = 2$ at any point $(1,1)$?

15. Find the asymptotes of $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.

16. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ of which one root is $1 - \sqrt{5}$.

17. Determine completely the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

18. Show that the equation $3x^4 - 8x^3 - 6x^2 + 24x - 7 = 0$ has one positive, one negative and two imaginary roots.

PART C

ANSWER ANY TWO QUESTIONS $\quad (2 * 20 = 40\ marks)$

19. a) If $y = \sin(m\sin^{-1} x)$, prove that $(1 - x^2)\,y_2 - xy_1 + m^2 y = 0$ and $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)\,y_n = 0$.

   b) Find the angle of intersection of the cardioids $\quad r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ **(10+10)**

20. Find the evolute of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

21. a) Show that if the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P. then $2p^3 - 9pq + 27r = 0$.

   b) Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$. $\qquad$ **(10+10)**

22. Using Horner's method, find the real root of the equation $x^3 - 3x + 1 = 0$.

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