LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **STATISTICS**

SECONDSEMESTER – APRIL 2018

MT 2906- REAL ANALYSIS AND LINEAR ALGEBRA

Date: 19-04-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

Answer **ALL** the questions.

1. a) Find $n_0 \in N$ such that $\left|\frac{n}{n+2} - 1\right| < \frac{1}{3}$ and find the limit of $\left\{\frac{n}{n+2}\right\}$. (5)

OR

b) If $\sum a_n$ is a convergent series then prove that $\lim_{n\to\infty} a_n = 0$. (5)

c) (i) If $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then prove that $\lim_{n\to\infty}(s_n + t_n) = (L + M)$.

(ii) If $\{a_n\}$ is a decreasing sequence of positive terms converging to zero then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ converges. (5+10)

OR

d) (i) Let $\sum a_n$ be a divergent series of positive numbers. Then prove that there is a sequence $\{\varepsilon_n\}$ of positive numbers which converges to zero but $\sum \varepsilon_n a_n$ diverges.

(ii) Determine the convergence or divergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \cdots$. (10+5)

2) a) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} f(x) = M$ then prove that $L = M$. (5)

OR

b) If the real valued function f is differentiable at the point $a \in R$ then prove that f is continuous at 'a'. (5)

c) (i) Prove that a real valued function f defined in a neighbourhood of a point 'a' is continuous at 'a' if and only if for every sequence $\{x_n\}$ in the domain of f converging to 'a', the sequence $\{f(x_n)\}$ converges to $f(a)$.

(ii) State and prove mean value theorem for derivatives. (8+7)

OR

d) (i) State and prove Taylor's Formula.

(ii) Define continuity, jump discontinuity and removable discontinuity. (9+6)

3. a) For any partition P of $[a, b]$, prove that $m[f; P](b - a) \le L[f; P] \le U[f; P] \le M[f; P](b - a)$. (5)

OR

b) State and prove second mean value theorem for integrals. (5)

(i) Let f be bounded function on the closed bounded interval $[a,b]$ then prove that f is Riemann integrable if and only if for every $\varepsilon > 0$ there exists a subdivision P of $[a,b]$ such that $U[f;P] - L[f;P] < \varepsilon$.

(ii) If f is monotone on $[a,b]$ then prove that f is Riemann integrable on $[a,b]$. (10+5)

OR

d) (i) State and prove First Fundamental theorem of Calculus.

(ii) If f is continuous function on the closed bounded interval $[a,b]$ and if $\varphi'(x) = f(x)$ for $x \in [a,b]$ then prove that $\int_a^b f(x)dx = \varphi(b) - \varphi(a)$. (10+5)

4. a) Show that the vectors $\{1,2,3\}$ and $\{3,2,1\}$ are linearly independent over the field of rational numbers. (5)

OR

b) If the kn-vectors $A_1, A_2, \dots, A_k$ are linearly independent but the vectors $A_1, A_2, \dots, A_k, B$ are linearly dependent then prove that B is a linear combination of $A_1, A_2, \dots, A_k$.
. (5)

c) (i) If the linear system of m equations in n unknowns $AX + B = 0$ is consistent then prove that a complete solution is given by a complete solution of the corresponding homogeneous system $AX = 0$ plus any particular solution of $AX + B = 0$.

(ii) If the kn-vectors $A_1, A_2, \dots, A_k$ are linearly independent then prove that any $k+1$ linear combinations of these n-vectors are linearly dependent. (8+7)

OR

d) (i) Let V_n be a vector space over F, not consisting of the zero vector alone then prove that V_n contains atleast one set of linearly independent vectors $A_1, A_2, \dots, A_k$ such that the collection of all linear combinations X of the form $X = t_1A_1 + t_2A_2 + \cdots + t_kA_k$ where $t's$ are arbitrary scalars from F, is precisely V_n. Moreover, prove that the integer k is uniquely determined for each V_n.

(ii) Find the complete solution of non-homogeneous system $x_1 - x_2 + 2x_3 = 1$ and $2x_1 + x_2 - x_3 = 2$. . (10+5)

5 a) Apply Gram Schmidt orthonormalization process to the vectors $(1,0,1), (1,0,-1), (0,3,4)$ to obtain an orthonormal basis for R^3. (5)

OR

b) Find the characteristic roots and their corresponding vectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. (5)

c) Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_1x_3$ to canonical form through an orthogonal transformation. (15)

OR

d) Explain the process of reduction to diagonal form and hence reduce the matrix

$A = \begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$ to its diagonal form. (15)

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